2 Mechanics

2.1 Motion

Answers to 2.1 Motion (Skills worksheet)

1) **Time:** 16.8 s; **Velocity:** 53.1 ms^{-1}

2) **Speed:** 324 m

3) **Distance:** 200 m; **Average speed:** 9.4 ms^{-1}

4) The truck arrives 0.104 hours earlier than the car.

5) **Time:** 40 s

6) They collide 3.3 m from the starting position of the first ball.

7) **Time:** 5.0 s

8)
   a) **Acceleration:** 2 ms^{-2}
   b) **Displacement:** 160 m
   c) **Average velocity:** 11.4 ms^{-1}

9) **Times:** 0.18 s and 1.1 s

10) **Time:** 3.8 s

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Answers to 2.2 Forces (Skills worksheet)

1) **Force:** 14 N
2) **Force:** 45 N
3) **Mass:** 9.7 kg
4) **Coefficient of dynamic friction:** 0.206
5) Yes, it will slide
6) **Acceleration:** 0.11ms^{-2} [S24^{o}W]

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2.3 Work, energy and power

Answers to 2.3 Energy transformations (Skills worksheet)

1)
   a) **Gravitational potential energy:** 353 J
   b) **Gravitational potential energy:** 235 J
   c) **Kinetic energy:** 118 J
   d) **Gravitational potential energy:** 177 J
   e) **Kinetic energy:** 177 J
   f) **Gravitational potential energy:** 0 J
   g) **Kinetic energy:** 353 J

2) The total is always the same.

3) **Speed:** 0.86 ms^{-1}

4) **Work:** 460 J

5) **Height:** 0.90 m

6) **Speed:** 15 ms^{-1}

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Answers to 2.3 Work, power and efficiency (Skills worksheet)

1) **Work:** 2200 J; **Power:** 180 W

2) **Work:** 24 J; **Velocity:** 2.0 ms^{-1}

3) **Output power:** 3200 W

4) **Efficiency:** 85%

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Answers to 2.4 Momentum and impulse (Skills worksheet)

1) **Impulse:** 71 000 Ns
2) **Velocity:** 0.4 ms^{-1}
3) **Velocity:** 3.7 ms^{-1}
4) **Mass:** 23 kg; **Velocity:** 1.8 ms^{-1}
5) **Velocity:** 0.37 ms^{-1} in the opposite direction that he threw the rocks.

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Constant acceleration

1. A body travelling at $5ms^{-1}$ accelerates at $20ms^{-2}$ for 100m. What will its final velocity be?

Use $v^2 = u^2 + 2as = 5^2 + 2 \times 20 \times 100$

$v = \sqrt{4025} = \pm 63.4 ms^{-1}$ $+63.4 ms^{-1}$

$5ms^{-1}$ $v ms^{-1}$ must be +ve

2. A body travelling at $2ms^{-1}$ accelerates to $8ms^{-1}$ in 2s. Find its displacement.

use $s = \left(\frac{u+v}{2}\right)t = \left(\frac{2+8}{2}\right) \times 2 = 10m$

3. A body is thrown downwards with a velocity of $10ms^{-1}$. Assuming the acceleration due to gravity is $10ms^{-2}$ calculate its displacement after 2s?

$10ms^{-1}$

$10ms^{-2}$

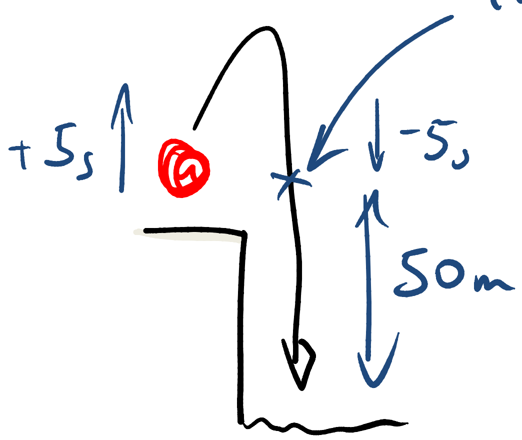
down is +ve

use $s = ut + \frac{1}{2}at^2$

$= 10 \times 2 + \frac{1}{2} 10 \times 2^2$

$= 20 + 20 = 40$ m

4. A boy standing on the edge of a cliff throws a stone upwards with a velocity of $5ms^{-1}$. If the cliff is 50m high how much time does it take for the stone to hit the bottom of the cliff?



time to here from $a = \frac{v-u}{t}$

$t = \frac{v-u}{a} = \frac{-5-5}{10} = 1s$

final velocity $v^2 = u^2 + 2as$

$= (-5)^2 + 2 \cdot -10 \cdot -50$

$= 25 + 1000 = 1025$

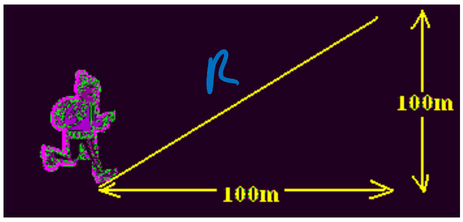
$v = 32 ms^{-1}$

time to bottom from $a = \frac{v-u}{t} \Rightarrow t = \frac{v-u}{a}$

$= \frac{-32 - -5}{-10} = 2.7s$ Total time $= 1 + 2.7 = 3.7s$

Distance, Displacement and Speed

1. A person walks up a hill as shown in the diagram.



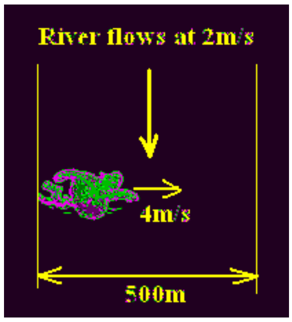
a) How many meters does the person walk?

Using pythagoras

$R^2 = 100^2 + 100^2 \Rightarrow R = \sqrt{20000}$

$= 141\text{ m}$

2. A person swims across a river as shown in the diagram.



a) How many seconds does the swimmer take to cross the river?

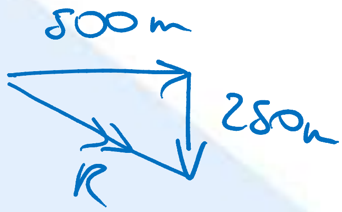
$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{500}{4}$

$\Rightarrow t = 125\text{ s}$

b) How far downstream will the swimmer be swept in the time taken to cross the river?

$s = ut \Rightarrow s = 2 \times 125$

$= 250\text{m}$



c) What is the final displacement of the swimmer?

add displacement vectors

$R = \sqrt{500^2 + 250^2} = 560\text{ m}$

3. The distance from Flekke to Dale is 6km on the line shown on the map. The direction is 20° East of North.

a) How far to the North of Flekke is Dale?

(Flekke is where I live)

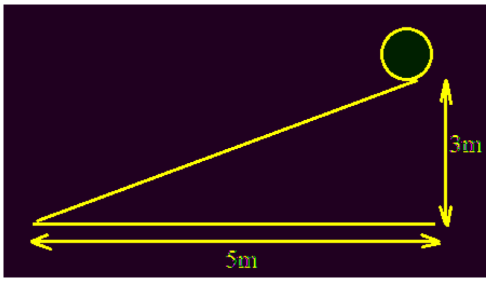
Find N component (next to angle)

$- 6 \times \cos 20° = 5.6\text{ km}$

Energy and Power

1. A 2kg ball rolls down the slope shown. Calculate the loss of PE.

$$mg\Delta h = 2 \times 10 \times 3$$
$$= 60J$$

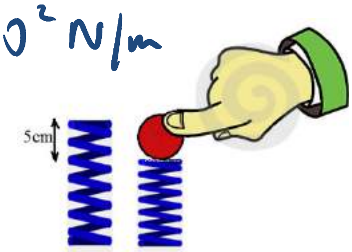


2. A 2kg ball is pushed onto a spring of spring constant 100N/cm as shown. (It's a very strong finger by the way)

100×10^2 N/m

(a) Calculate elastic PE stored in the spring

$$EPE = \tfrac{1}{2} kx^2$$
$$= \tfrac{1}{2}\, 10^4 \times 0.05^2$$
$$= 12.5J$$



(b) Calculate the maximum height reached by the ball

Cons. of energy - $mgh = \tfrac{1}{2}kx^2$

$$h = \frac{12.5}{2 \times 10} = 0.625\ m$$

3. A machine is used to lift a 50kg mass 4m vertically in 10s.

(a) Calculate the increase in PE

$$mgh = 50 \times 10 \times 4 = 2000J$$

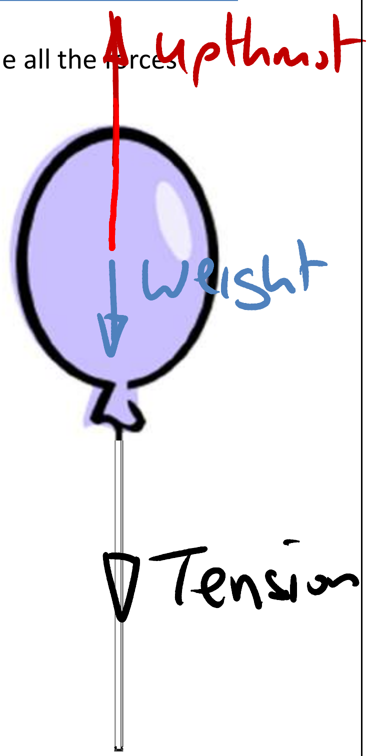
(b) Calculate the power of the machine.

$$Power = \frac{E}{time} = \frac{2000}{10}$$
$$= 200W$$

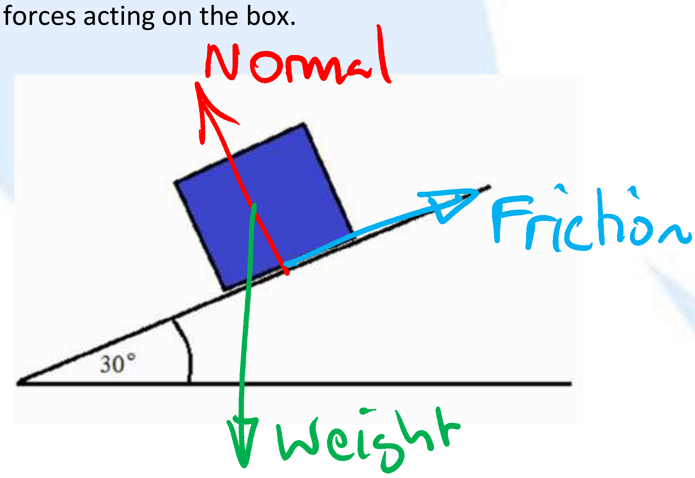
Formulae
KE = $\tfrac{1}{2}mv^2$
PE=mgh
EPE=$\tfrac{1}{2}kx^2$
P=E/t

Forces

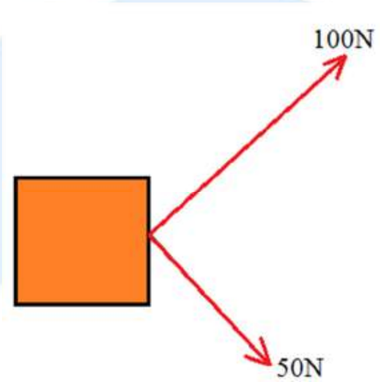
1. A Helium balloon is held stationary with a string as shown. Draw and name all the forces acting on the balloon.



2. A box rests on a slope as shown in the diagram. Draw and label the forces acting on the box.



3. Two perpendicular ropes are use to pull a load along the ground as shown in the diagram. Calculate the resultant force.



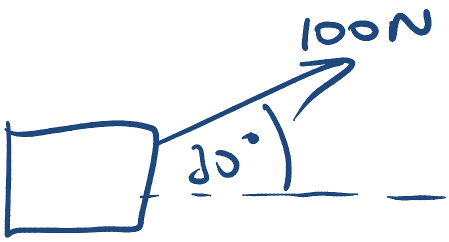
Using pythagoras

$$F = \sqrt{100^2 + 50^2} = 112\,N$$

4. A Force acts at a 30° angle to the horizontal as shown. Calculate the Horizontal component of the force.

$$F_H = 100 \cdot \cos 30^\circ$$

$$= 87\,N$$



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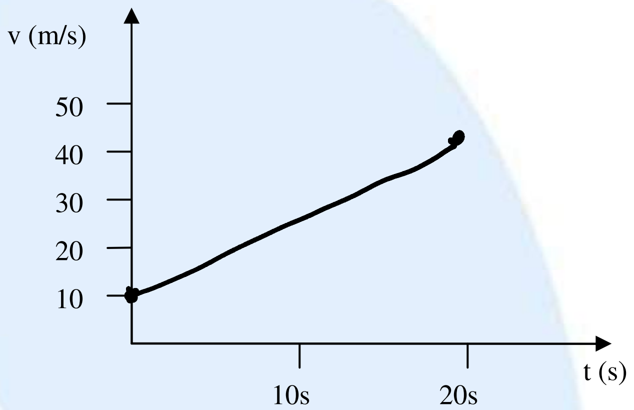
Graphs of Motion

1. A car travelling at 10m/s accelerates at a constant rate for 20s until its velocity is 40m/s.



a. Using the axis above draw v-t graph for this motion.
b. Use the graph to calculate the distance traveled

$s =$ area under graph $= \frac{1}{2}(10+40) \times 20$

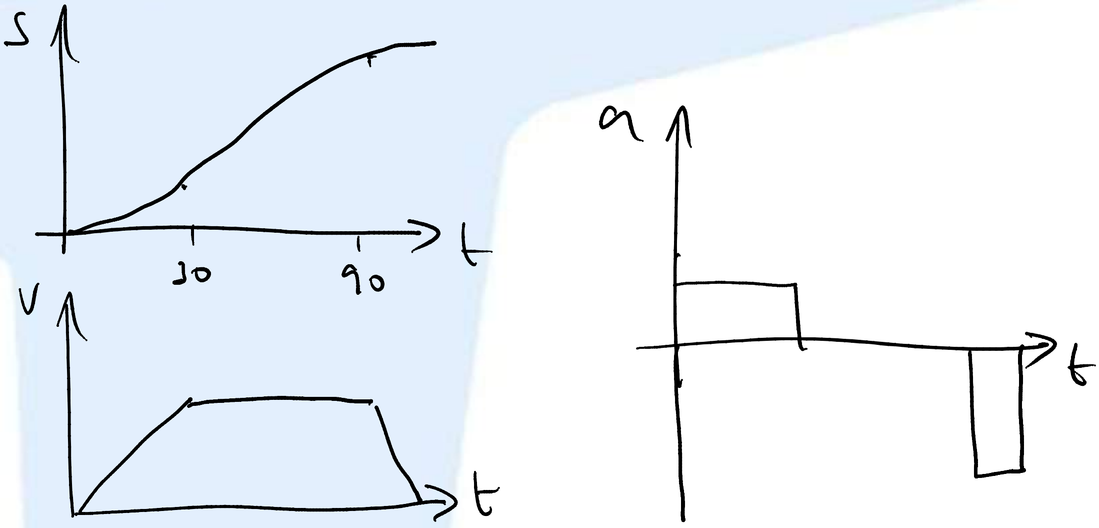
$= 500m$

c. Use the graph to calculate the acceleration.

$a =$ gradient $= \frac{40-10}{20} = 1.5 ms^{-2}$

2. The motion of a car travelling a long a straight road can be split into 3 stages:
   (i) 30s of constant acceleration from rest.
   (ii) 60s of constant velocity
   (iii) 10s of braking to stop

Draw displacement – time, velocity – time and acceleration – time graphs for this motion



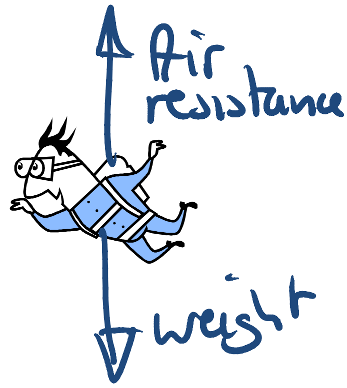
Newton's 1st

1. State Newton's 1st law.

A body remains at rest or moving with constant velocity unless acted upon by an unbalanced force.

2. A freefall parachutist falls with constant velocity.

(a) Label the Forces acting on the parachutist in the diagram.



(b) What can you deduce about the size of these forces?

If constant vel – forces are balanced.

3. A plastic football is held underneath the water in a swimming pool and released. The mass of the ball is 500g and the upthrust it experiences is 40N.

(a) Draw the forces on the ball in the diagram.

(b) What is the resultant force?

W = 5N resultant = 40 − 5 = 35N

(c) What can you deduce about the motion of the ball?

The ball will accelerate since forces not balanced.

4. Calculate the momentum of a 1200kg car travelling at $20ms^{-1}$.

$p = mv = 1200 \times 20 = 24 \times 10^4$ Ns

Formulae
P=mv

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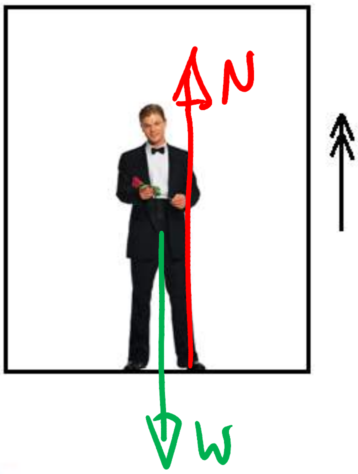
Newton's 2nd

1. State Newton's 2nd law of motion.

The rate of change of momentum of a body is directly proportional to the resultant force acting on it and takes place in the same direction.

2. A man is standing in a lift that is accelerating upwards at a rate of $2ms^{-2}$.

(a) Draw the forces acting on the man



(b) If the mass of the man is 60kg calculate the Normal Force acting on his feet.

$N - W = ma$

$N = ma + W$

$= 60 \times 2 + 60 \times 10$

$= 720\,N$

3. Forces act on the 50kg mass as shown

(a) Calculate the acceleration of the mass.

Resultant $F = 600N$

$F = ma \quad a = \frac{F}{m} = \frac{600}{50} = 12 ms^{-2}$

(b) If the original momentum is zero calculate the momentum of the box after 2s.

$F =$ rate of change of momentum

$= \frac{\Delta mv}{t}$

$\Delta m = Ft = 600 \times 2 = 1200Ns$

or $a = \frac{\Delta v}{t} \quad \Delta v = at = 12 \times 2$

$\Delta p = m\Delta v = 24 \times 50 = 1200Ns$

Formulae
F=ma

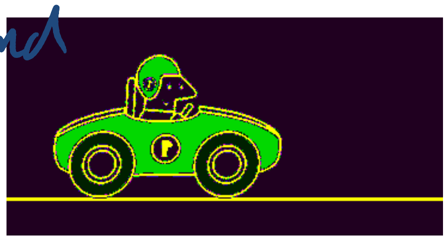
Newton's 3rd

1. State Newton's 3rd law of motion.

If body A exerts a force on body B then body B will exert an equal and opposite force on body A.

2. When a car accelerates the friction of the tyres push the ground backwards. Use Newton's 3rd law to deduce what happens to the car.

If the car pushes the ground backwards then the ground will push the car forwards



3. Two balls collide as shown in the diagram. Use the principle of conservation of momentum to calculate the velocity of the big ball.

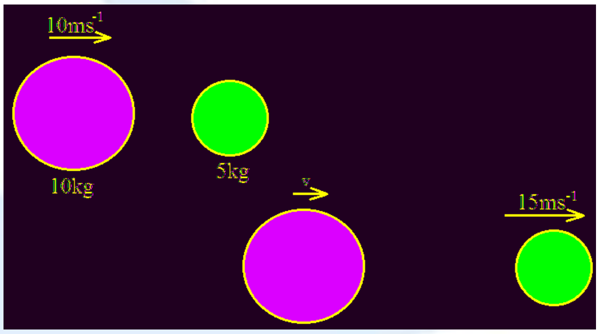
momt. before = momt. after

$10 \times 10 + 5 \times 0 = 10 \times v + 5 \times 15$

$100 = 10v + 75$

$10v = 25$

$v = 2.5\ ms^{-1}$



4. Calculate the impulse of the big ball in the previous question.

Impulse = change in momt.

= final momt. − initial momt.

$= 10 \times 2.5 - 10 \times 10$

$= 25 - 100$

$= {}^{-}75$ Ns

Formulae
p=mv
Impulse = Δmv

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Projectiles

1. A ball is projected as shown in the diagram.

parabolic

Formulae
$s = ut + \frac{1}{2}at^2$
$v^2 = u^2 + 2as$
$a = \frac{v-u}{t}$
$s = \frac{(u+v)}{2}t$

(a) Draw a line showing the possible path of the ball.

(b) Calculate the vertical component of the balls velocity.

$10\sin 30° = 5\,ms^{-1}$

(c) Using your answer to (b) calculate the time of flight.

$a = \frac{v-u}{t}$ $\quad t = \frac{v-u}{a} = \frac{-5-5}{-10} = 1s$

(d) Calculate the horizontal velocity of the ball.

$10\cos 30° = 8{\cdot}7\,ms^{-1}$

(e) Using your answer to (c) and (d) calculate the balls range.

$v = \frac{s}{t}$ $\quad s = vt = 8{\cdot}7 \times 1 = 8{\cdot}7m$

2. The ball above is projected with the same angle and velocity from a 10m high sea cliff. Calculate the time taken for the ball to hit the sea.

vertically $u = 5ms^{-1}$

$a = -10ms^{-2}$

$s = -10$

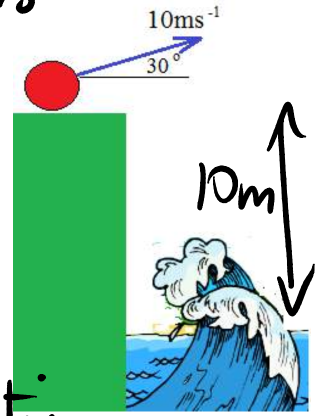
$t = ?$

$v = ?$

$v^2 = u^2 + 2as$

$= 25 + 200 = 225$

$v = -15ms^{-1}$



$a = \frac{v-u}{t}$ $\quad t = \frac{v-u}{a} = \frac{-15-5}{-10}$

$= 2s$

or use $s = ut + \frac{1}{2}at^2$ & solve quadratic.

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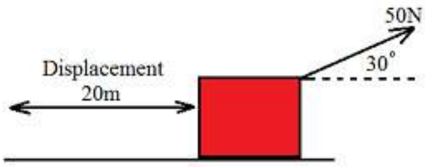
Work and Energy

1. Define work.

Work is done when the point of application of a force moves in the direction of the force.

2. Calculate the amount of work done by the force in the following situations where the box is travelling to the right.

(a)

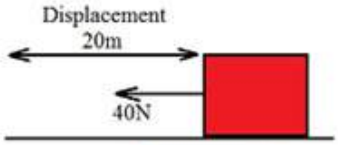


$W = F\cos\theta . x$

$= 50.\cos 30 . 20$

$= 866J$

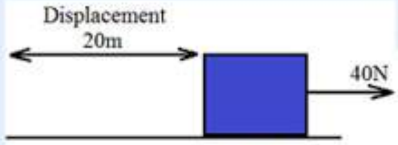
(b)



$W = {}^{-}40 . 20$

$= -800J$

3. A body is accelerated by a force as shown in the diagram. Calculate the final velocity of the body.



10kg

$W = 40.20 = 800J$

$\Delta KE = \Delta W \Rightarrow 800 = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2.800}{10}} = 12.6ms^{-1}$

4. Calculate the KE of a 1000kg car travelling at $30ms^{-1}$.

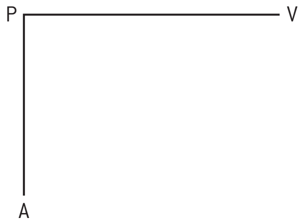
$KE = \frac{1}{2}mv^2 = \frac{1}{2}.1000.30^2$

$= 4.5\times10^5 J$

Formulae $W=Fx\cos\theta$ $KE = \frac{1}{2}mv^2$

Solutions for Topic 2 – Mechanics

1. **a)**



b) horizontal speed $= 15 \times \cos 45 = 10.6\ \text{m s}^{-1}$

vertical speed $= 15 \times \sin 45 = 10.6\ \text{m s}^{-1}$ upwards

$v^2 = u^2 + 2as$; $v^2 = 10.6^2 + 2 \times 9.8 \times 25 = 112 + 490 = 602$

$v = \pm\ 24.5\ \text{m s}^{-1}$ (positive value is correct one to use)

so speed is $\sqrt{10.6^2 + 24.5^2}$

$= 27\ \text{m s}^{-1}$

2. **a)** **(i)** $h = \dfrac{v^2}{2g} = 3.2\ \text{m}$

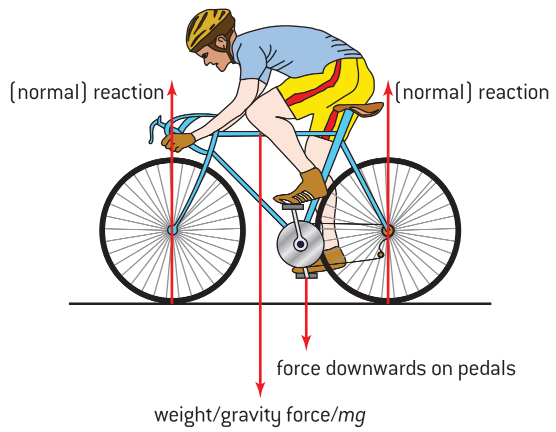
(ii) $t = \dfrac{u}{g} = 0.80\ \text{s}$

b) time to go from top of cliff to the sea $= 3.0 - 1.6 = 1.4\ \text{s}$
$s = 8.0 \times 1.4 + 5.0 \times (1.4)^2 = 21\ \text{m}$;

3. travels vertically 1.25 m in 0.5 s;
$g = \dfrac{2s}{t^2}$
to give $g = 10\ (\pm 1)\ \text{m s}^{-2}$

4. **a)** **(i)** Zero

(ii)



(iii) The drag force is equal to the forward force; the net force is zero and therefore the acceleration is zero.

b) **(i)** acceleration $= \dfrac{\text{resistive force}}{\text{mass}} = \dfrac{40}{70} = 0.57\ \text{m s}^{-2}$

(ii) $v^2 = u^2 + 2as$; $0 = 64 - (2 \times 0.57 \times s)$; $s = 56\ \text{m}$

(iii) air resistance ***or*** bearing friction ***or*** effectiveness of brakes depends on speed; air resistance reduced as speed drops, estimate will be too low, stopping distance will be further

5. The net force on the car is $0.3 \times 1000 = 300\ \text{N}$. There is an additional drag force of 500 N.
$T = 300 + 500 = 800\ \text{N}$.

6. $T_1 \sin 60 = T_2 \sin 30$

$T_1 \cos 60 + T_2 \cos 30 = 3800$

$T_1 = 1900\ \text{N}$; $T_2 = 3300\ \text{N}$

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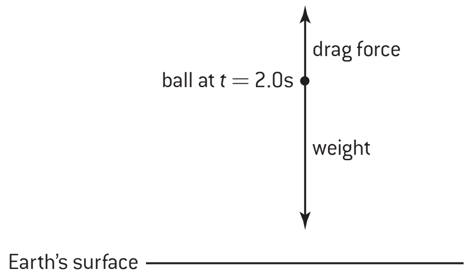
7. **a)** power is 0.66 kW (read off from graph)

b) power = frictional force × speed

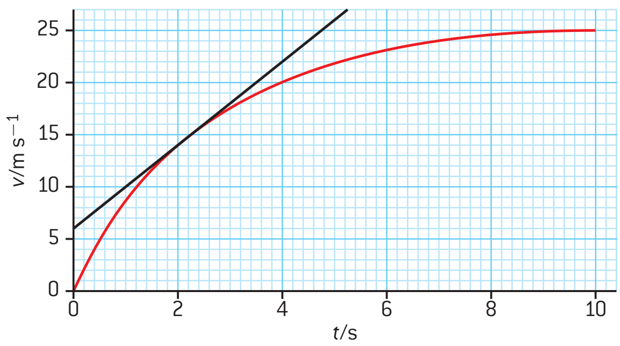
force $= \dfrac{660}{2} = 330$ N

8. **a)** use the area under the graph as this is $v \times t$

b) **(i)**



(ii)



the acceleration of the ball is equal to the gradient of the graph gradient $= \dfrac{25 - 6}{4.8 - 0}$

$= 4.0$ m s^{-2}

(iii) The net force on the ball is 2 N, the weight is 4.9 N, so the difference between these is the magnitude of the drag force = 2.9 N.

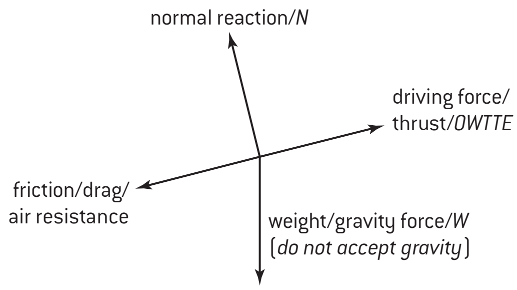
(iv) At 5.0 s the gradient is smaller and therefore the acceleration is less than at 2.0 s. The weight is constant and therefore the drag force is greater.

c) gain in kinetic energy $= \dfrac{1}{2} \times 0.5 \times 25^2 = 156$ J

loss in gravitational potential energy $= 0.5 \times 9.8 \times 190 = 931$ J

change (loss) in energy $= 931 - 156 = 775$ J

9. **a)** **(i)**



(ii) zero

b) input power $= \dfrac{\text{output power}}{\text{efficiency}} = \dfrac{70}{0.35}$

$= 200$ kW

c) height gained in 1 s $= 6.2 \sin(6) = 0.648$ (m)

rate of change of PE $= 8.5 \times 10^3 \times 9.81 \times 0.648$

$= 5.4 \times 10^4$ W

d) $F = \left(\dfrac{p}{v}\right) = \dfrac{1.6 \times 10^4}{6.2}$

$= 2.6$ kN

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10. a) **(i)** momentum before $= 800 \times 5 = 4\,000$ N s

momentum after $= 2\,000v$

conservation of momentum gives $v = 2.0$ m s^{-1}

(ii) KE before $= 400 \times 25 = 10\,000$ J KE after $= 1\,000 \times 4 = 4\,000$ J
loss in KE $= 6\,000$ J;

b) transformed / changed into heat (internal energy) and sound

11. a) momentum of object $= 2 \times 10^3 \times 6.0$

momentum after collision $= 2.4 \times 10^3 \times v$

use conservation of momentum, $2 \times 10^3 \times 6.0 = 2.4 \times 10^3 \times v$

$v = 5.0$ m s^{-1}

b) KE of object and bar + change in PE $= 0.5 \times 2.4 \times 10^3 \times 25 + 2.4 \times 10^3 \times 10 \times 0.75$
use $\Delta E = Fd$, $4.8 \times 10^4 = F \times 0.75$
$F = 64$ kN

12. a) time $= \dfrac{81}{2.2 \times 10^{-25} \times 77 \times 10^{18}} = 4.8 \times 10^7$ s

b) rate of change of momentum of the xenon atoms

$= 77 \times 10^{18} \times 2.2 \times 10^{-25} \times 3.0 \times 10^4$

$= 0.51$ N

$=$ mass $\times$ acceleration

where mass $= (540 + 81)$ kg

acceleration of spaceship $= \dfrac{0.51}{621}$

$= 8.2 \times 10^{-4}$ m s^{-2}

c) $a = \dfrac{F}{m}$

since m is decreasing with time, then a will be increasing with time

d) change in speed = area under graph

$= (8.0 \times 4.8) \times 10^2 + \frac{1}{2}(4.8 \times 1.4) \times 10^2$

final speed $= (8.0 \times 4.8) \times 10^2 + \frac{1}{2}(4.8 \times 1.4) \times 10^2 + 1.2 \times 10^3\ 5.4 \times 10^3$ m s^{-1}

13. a) centripetal force $= \dfrac{(350 \times 2.6^2)}{5.8} = 410$ N

tension $= 410 + (350 \times 9.8) = 3800$ N

b) idea of use of area under graph

distance $= \frac{1}{2} \times 0.15 \times 2.6$

$= 0.195$ m

c) idea of momentum as mv

total change $(= 2.6 \times 350) = 910$ N s

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1. A (1)

2. A (1)

3. D (1)

4. C (1)

5. B (1)

6. D (1)

7. D (1)

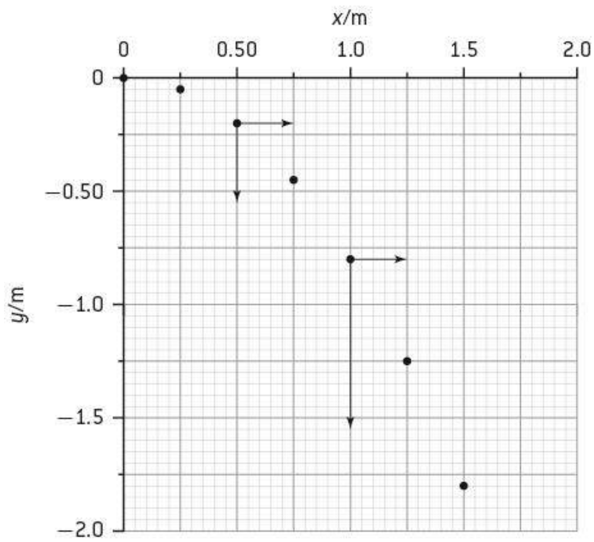
8. C (1)

9. A (1)

10. B (1)

Total: 10 marks

1. a)



Mark both together.

V_H: horizontal arrows equal in length;

V_V: two vertical arrows, the one at 1.0 m noticeably longer than the one at 0.5 m; (2)

If arrows correct, but wrong points award (1) only.

b) curve that goes through all data points;

stops at y = 1.8 m as this is the height of the wall;

from graph d = 1.5 (±0.1) m; (3)

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c) travels vertically 1.8 m in 0.6 s /1.25 m in 0.5 s;

$g = (2s / t^2)$;

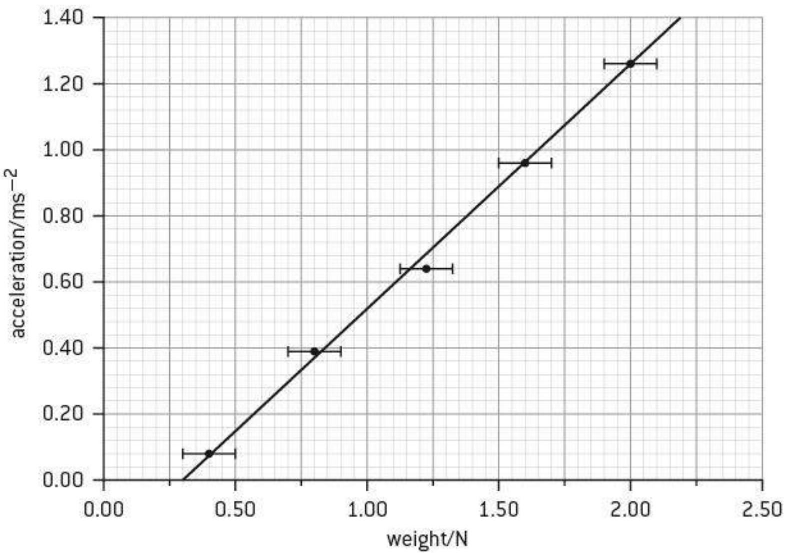
to give g = 10 (± 1) m s^{-2}; (3)

Award (2) marks maximum for any time shorter than 0.5 s.

Total: 8 marks

2. a) a straight line; through the origin; (2)

b) any straight line; that fits within all the error bars; (2)



c) (i) a *systematic* error is when every data point deviates from the "correct" value;

by the same fixed amount as seen by intercept on graph; (2)

Accept answers that explain by giving an example of a possible systematic error (e.g. friction).

(ii) 0.3 N; (1)

Accept 0.25 N to 0.35 N; watch for use of wrong axis.

Total: 7 marks

3. a) (i) attempt using principle of moments (even if in error);

F x 1.2 = 600 x 0.4;

therefore F = 200 N; (3)

(ii) resultant force = zero, therefore reaction = 600 – 200 = 400 N;

up; (2)

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b) (i) correct use of $F_{horizontal}$ = 260 cos40 or 260 sin50;

to give $F_{horizontal}$ = 1992 N ≈ 200 N; (2)

(ii) realization that resultant force is zero (constant velocity);

so friction = $F_{horizontal}$ ≈ 200 N; (2)

Total: 9 marks

4. a) mass x velocity; (1)

b) (i) momentum before = 800 x 5 = 4000 N s;

momentum after = 2000ν;

conservation of momentum gives ν = 2.0 m s^{-1}; (3)

(ii) KE before = 400 x 25 = 10 000 J

KE after = 1000 x 4 = 4000 J;

loss in KE = 6000 J; (2)

Total: 6 marks

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Answers to exam-style questions

Topic 2

Where appropriate, 1 ✓ = 1 mark

1 D

2 C

3 C

4 D

5 A

6 D

7 D

8 A

9 C

10 A

11 a i The equation applies to straight line motion with acceleration g. Neither condition is satisfied here. ✓

ii This equation is the result of energy conservation so it does apply since there are no frictional forces present. ✓

b From $v=\sqrt{2gh}$ we find $h=\frac{v^2}{2g}=\frac{4.8^2}{2\times 9.81}=1.174\approx 1.2$ m. ✓

c i The kinetic energy at B is $E=\frac{1}{2}mv^2=\frac{1}{2}\times 25\times 4.8^2=28.8$ J. ✓

The frictional force is $f=\mu_K N=\mu_K mg=0.45\times 25\times 9.81=110.36$ N and so the work done by this force is the change in the kinetic energy of the block, and so $110.36\times d=28.8\Rightarrow d=0.261\approx 0.26$ m. ✓

ii The deceleration is $\frac{f}{\mu}=\frac{110.36}{25}=4.41\ \text{m s}^{-2}$, ✓

and so $0=4.8-4.41\times t$ giving 1.1 s for the time. ✓

d The speed at B is independent of the mass. ✓

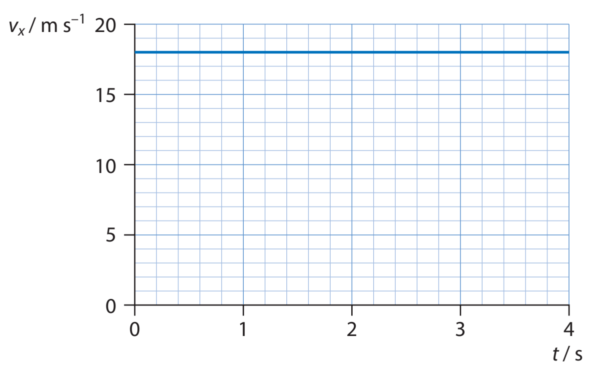
$fd=\frac{1}{2}mv^2\Rightarrow \mu_K mgd=\frac{1}{2}mv^2\Rightarrow d=\frac{v^2}{2\mu_K}$, ✓

and so the distance is also independent of the mass. ✓

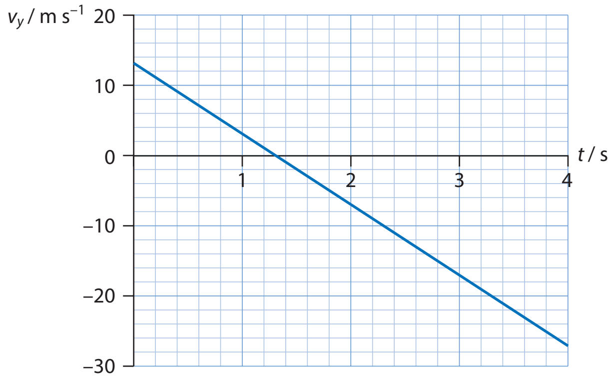
12 a i $v_x=v\cos\theta=22\times\cos 35°=18.0\approx 18\ \text{m s}^{-1}$ ✓

$v_y=v\sin\theta=22\times\sin 35°=12.6\approx 13\ \text{m s}^{-1}$ ✓

ii Graph as shown. ✓



Graph as shown. ✓



b i At maximum height: $v_y^2 = 0 = u_y^2 - 2gy$. ✓

$y = \dfrac{u_y^2}{2g}$ ✓

and so $y = \dfrac{12.6^2}{2 \times 9.8} = 8.1$ m ✓

OR

$v_y = 0 = v\sin\theta - gt$ $12.6 - 9.8t = 0$ ✓

so $t = 1.29$ s ✓

Hence $y = 12.6 \times 1.29 - \dfrac{1}{2} \times 9.8 \times 1.29^2 = 8.1$ m ✓

ii The force is the weight, i.e. $F = 0.20 \times 9.8 = 1.96 \approx 2.0$ N. ✓

c i $\dfrac{1}{2}mu^2 + mgh = \dfrac{1}{2}mv^2$ hence $v = \sqrt{u^2 + 2gh}$ ✓

$v = \sqrt{u^2 + 2gh} = \sqrt{22^2 + 2 \times 9.8 \times 32} = 33.3 \approx 32$ m s^{-1} ✓

ii $v^2 = v_x^2 + v_y^2 \Rightarrow v_y = -\sqrt{v_x^2 - v_x^2} = -\sqrt{33.3^2 - 18.0^2} = -28.0$ m s^{-1} ✓

Now $v_y = u_y \sin\theta - gt$ so $-28.0 = 12.6 - 9.8 \times t$ hence $t = 4.1$ s ✓

d i Smaller height. ✓

Smaller range. ✓

Steeper impact angle. ✓

ii The angle is steeper because the horizontal velocity component tends to become zero. ✓
Whereas the vertical tends to attain terminal speed and so a constant value. ✓

13 a i In 1 second the mass of air that will move down is $\rho(\pi R^2 v)$. ✓

The change of its momentum in this second is $\rho(\pi R^2 v)v = \rho\pi R^2 v^2$. ✓

And from $F = \dfrac{\Delta p}{\Delta t}$ this is the force. ✓

ii $\rho\pi R^2 v^2 = mg$ ✓

And so $v = \sqrt{\dfrac{mg}{\rho\pi R^2}} = \sqrt{\dfrac{0.30 \times 9.8}{1.2 \times \pi \times 0.25^2}} = 3.53 \approx 3.5 \text{ m s}^{-1}$. ✓

b The power is $P = Fv$ where $F = \rho\pi R^2 v^2$ is the force pushing down on the air and so $P = \rho\pi R^2 v^2$. ✓
So $P = 1.2 \times \pi \times 0.25^2 \times 3.53^2 = 2.936 \approx 3.0 \text{ W}$ ✓

c i From $F = \rho\pi R^2 v^2$ the force is now 4 times as large, i.e. $4mg$ and so the **net** force on the helicopter is $3mg$. ✓

And so the acceleration is constant at $3g$. Hence $s = \dfrac{1}{2} \times 3g \times t^2 \Rightarrow t = \sqrt{\dfrac{2s}{3g}} \approx 0.90$ s. ✓

ii $v = 3gt = \sqrt{\dfrac{2s}{3g}}$ ✓

$v \approx 26 \text{ m s}^{-1}$ ✓

iii The work done by the rotor is $W = Fd = 4mgd = 4 \times 0.30 \times 9.8 \times 12 = 141$ J. ✓

14 a i The area is the impulse i.e. 2.0×10^3 N s. ✓

ii The average force is found from $\bar{F}\Delta t = 2.0 \times 10^3$ N s. ✓

And so $\bar{F} = \dfrac{2.0 \times 10^3}{0.20} = 1.0 \times 10^4$ N. ✓

Hence the average acceleration is $\bar{a} = \dfrac{1.0 \times 10^4}{8.0} = 1.25 \times 10^3 \text{ m s}^{-2}$. ✓

iii The final speed is $\bar{v} = \bar{a}t = 1.25 \times 10^3 \times 0.20 = 250 \text{ m s}^{-1}$. ✓

And so the average speed is 125 m s^{-1}. ✓

iv $s = \dfrac{1}{2}\bar{a}t^2 = \dfrac{1}{2} \times 1.25 \times 10^3 \times 0.20^2$ ✓

$s = 25$ m ✓

b i The final speed is $\bar{v} = \bar{a}t = 1.25 \times 10^3 \times 0.20$, ✓
$\bar{v} = 250 \text{ m s}^{-1}$. ✓

ii The kinetic energy is $E_K = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 8.0 \times 250^2$ ✓

$E_K = 2.5 \times 10^5$ J ✓

iii $P = \dfrac{E_K}{t} = \dfrac{2.5 \times 10^5}{0.20}$ ✓

$P = 1.25 \times 10^6$ W ✓

15 a i It is zero (because the velocity is constant). ✓

ii $F - mg\sin\theta - f = 0$ ✓

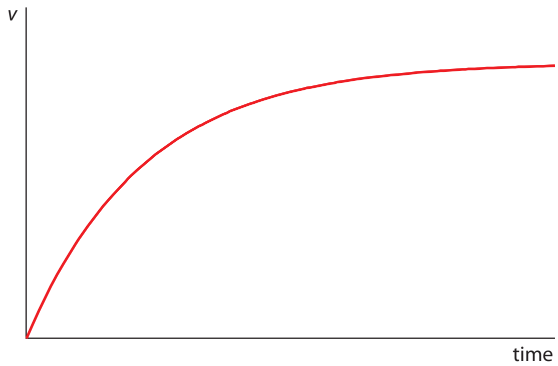
$F = mg\sin\theta + f = 1.4 \times 10^4 \times \sin 5.0° + 600$ ✓

$F = 1820$ N ✓

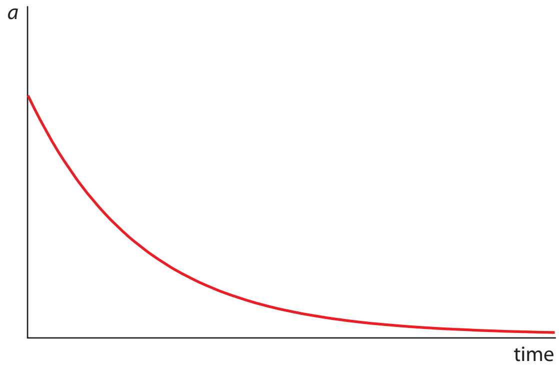
b The power used by the engine in pushing the car is $P = Fv = 1820 \times 6.2 = 1.13 \times 10^4$ W, ✓
$P = 11.3$ kW. ✓

The efficiency is then $e = \dfrac{11.3}{15} = 0.75$ ✓

c i Initial speed zero. ✓
Terminal speed. ✓



ii Initial acceleration not zero. ✓
And approaching zero. ✓



16 a i The change in momentum is $\Delta p = 0.090 \times (90 - 130)$, ✓
$\Delta p = -3.6$ N s. ✓

ii This is also the negative change in the momentum of the block and so $1.20v = 3.6$ N s giving $v = 3.0$ m s^{-1}. ✓

iii The initial kinetic energy is $E = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.090 \times 130^2 = 422.5$ J. ✓

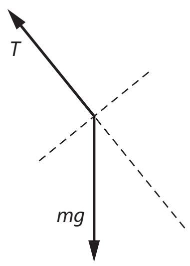
The final kinetic energy is $E = \frac{1}{2} \times 0.090 \times 90^2 + \frac{1}{2} \times 1.20 \times 3.0^2 = 369.9$ J. The change is then $\Delta E = 369.9 - 422.5 = -52.6 \approx -53$ J. ✓

b We have conservation of energy and so $\frac{1}{2} \times m \times 3.0^2 = m \times 9.8 \times h$ and so $h = 0.459$ m. ✓

But $h = L - L\cos\theta$ and so $0.459 = 0.80 \times (1 - \cos\theta)$ ✓

giving $\cos\theta = 0.426$ and so $\theta = 64.77° \approx 65°$ ✓

c i It is not because there is a net force on it. ✓



ii From the diagram, $T - mg\cos\theta = m\frac{v^2}{L}$. ✓

But $v = 0$ and so $T = mg\cos\theta = 1.20 \times 9.8 \times \cos 64.77° = 5.0$ N. ✓

$T = 5.0$ N. ✓

Topic 2.1a Kinematics Problems

Conceptual Questions

(These questions are not in an IB style but instead designed to check your understanding of the concept of this topic. You should try your best to appropriately communicate your answer using prose)

1. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.

   If an object is at the instant of reversing direction (like an object thrown upward, at the top of its path), it instantaneously has a zero velocity and a non-zero acceleration at the same time. A person at the exact bottom of a "bungee" cord plunge also has an instantaneous velocity of zero but a non-zero (upward) acceleration at the same time.

2. Can the velocity of an object be negative when its acceleration is positive? What about vice-versa?

   The velocity of an object can be negative when its acceleration is positive. If we define the positive direction to be to the right, then an object traveling to the left that is having a reduction in speed will have a negative velocity with a positive acceleration.

   If again we define the positive direction to be to the right, then an object traveling to the right that is having a reduction in speed will have a positive velocity and a negative acceleration.

3. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.

   Assume that north is the positive direction. If a car is moving south and gaining speed at an increasing rate, then the acceleration will be getting larger in magnitude. However, since the acceleration is directed southwards, the acceleration is negative, and is getting more negative. That is a decreasing acceleration as the speed increases.

   Another example would be an object falling WITH air resistance. As the object falls, it gains speed, the air resistance increases. As the air resistance increases, the acceleration of the falling object decreases, and it gains speed less quickly the longer it falls.

Topic 2.1b Kinematics Problems

Calculation-based Questions

1. A car accelerates from 13m/s to 25m/s in 6.0s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

[2 marks]

By definition, the acceleration is $a = \dfrac{v - v_0}{t} = \dfrac{25\,\text{m/s} - 13\,\text{m/s}}{6.0\ \text{s}} = \boxed{2.0\,\text{m/s}^2}$.

The distance of travel can be found from Eq. 2-11b.

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2 = (13\,\text{m/s})(6.0\ \text{s}) + \tfrac{1}{2}(2.0\,\text{m/s}^2)(6.0\ \text{s})^2 = \boxed{114\ \text{m}}$$

2. A car slows down from 23m/s to rest in a distance of 85m. What was the acceleration, assumed constant?

[1 mark]

The acceleration can be found from Eq. (2-11c).

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (23\,\text{m/s})^2}{2(85\ \text{m})} = \boxed{-3.1\,\text{m/s}^2}.$$

3. Estimate how long it took King Kong to fall straight down from the top of the Empire State Building (380m high) and his velocity just before he touched the ground. Ignore air resistance.

[2 marks]

Choose downward to be the positive direction, and take $y_0 = 0$ to be at the top of the Empire State Building. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\text{m/s}^2$.

(*a*) The elapsed time can be found from Eq. 2-11b, with *x* replaced by *y*.

$$y - y_0 = v_0 t + \tfrac{1}{2} a t^2 \quad \rightarrow \quad t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(380\ \text{m})}{9.80\,\text{m/s}^2}} = 8.806\,\text{s} \approx \boxed{8.8\ \text{s}}.$$

(*b*) The final velocity can be found from equation (2-11a).

$$v = v_0 + at = 0 + (9.80\,\text{m/s}^2)(8.806\ \text{s}) = \boxed{86\,\text{m/s}}$$

4. A stone is thrown vertically upward with a speed of 18.0m/s. How fast is it moving when it reaches a height of 11.0m and how long is required to reach this height? Why are there two answers?

[3 marks]

Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 18.0\,\mathrm{m/s}$, $a = -9.80\,\mathrm{m/s^2}$, and $y - y_0 = 11.0$ m .

(a) The velocity can be found from Eq, 2-11c, with x replaced by y.

$$v^2 = v_0^2 + 2a(y - y_0) = 0 \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2ay} = \pm\sqrt{(18.0\,\mathrm{m/s})^2 + 2(-9.80\,\mathrm{m/s^2})(11.0\ \mathrm{m})} = \pm 10.4\,\mathrm{m/s}$$

Thus the speed is $\boxed{|v| = 10.4\,\mathrm{m/s}}$

(b) The time to reach that height can be found from equation (2-11b).

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2(18.0\,\mathrm{m/s})}{-9.80\,\mathrm{m/s^2}}t + \frac{2(-11.0\ \mathrm{m})}{-9.80\,\mathrm{m/s^2}} = 0 \quad \rightarrow$$

$$t^2 - 3.673t + 2.245 = 0 \quad \rightarrow \quad \boxed{t = 2.90\ \mathrm{s}\ ,\ 0.775\ \mathrm{s}}$$

(c) $\boxed{\text{There are two times at which the object reaches that height}}$ – once on the way up $(t = 0.775\ \mathrm{s})$, and once on the way down $(t = 2.90\ \mathrm{s})$.

5. A stone is thrown vertically upward with a speed of 12.0m/s from the edge of a cliff 70.0m high. How long does it take to reach the bottom of the cliff and what is its speed before hitting? What was the total distance that it traveled? Ignore air resistance.

[3 marks]

Choose downward to be the positive direction, and $y_0 = 0$ to be at the top of the cliff. The initial velocity is $v_0 = -12.0\,\mathrm{m/s}$, the acceleration is $a = 9.80\,\mathrm{m/s^2}$, and the final location is $y = 70.0$ m .

(a) Using Eq. 2-11b and substituting y for x, we have

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow \left(4.9\,\mathrm{m/s^2}\right)t^2 - (12.0\,\mathrm{m/s})t - 70\ \mathrm{m} = 0 \rightarrow t = -2.749\ \mathrm{s}\ ,\ 5.198\ \mathrm{s}\ .$$

The positive answer is the physical answer: $\boxed{t = 5.20\ \mathrm{s}}$.

(b) Using Eq. 2-11a, we have $v = v_0 + at = -12.0\,\mathrm{m/s} + \left(9.80\,\mathrm{m/s^2}\right)(5.198\ \mathrm{s}) = \boxed{38.9\,\mathrm{m/s}}$.

(c) The total distance traveled will be the distance up plus the distance down. The distance down will be 70 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Then using Eq. 2-11c:

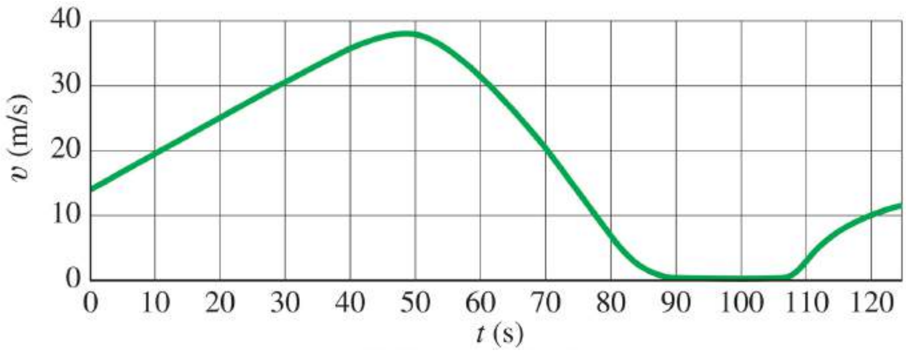
$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (-12.0\,\mathrm{m/s})^2}{2\left(9.80\,\mathrm{m/s^2}\right)} = -7.35\ \mathrm{m}\ .$$

Thus the distance up is 7.35 m, the distance down is 77.35 m, and the total distance traveled is $\boxed{84.7\ \mathrm{m}}$.

Topic 2.1b Kinematics Problems

Calculation-based Questions

1. The graph below shows the velocity of a train as a function of time.



   a. At what time was its velocity the greatest?
   b. During which periods, if any, was the velocity constant?
   c. During what periods, if any, was the acceleration constant?
   d. When was the magnitude of the acceleration the greatest?

[4 marks]

   Esimtate the distance the object travelled

   e. during the first minute and;
   f. in the second minute

[2 marks]

Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t \approx 48\ \text{s}}$.

(*b*) The indication of a constant velocity on a velocity-time graph is a slope of 0, which occurs from $\boxed{t = 90\ \text{s to } t \approx 108\ \text{s}}$.

(*c*) The indication of a constant acceleration on a velocity-time graph is a constant slope, which occurs from $\boxed{t = 0\ \text{s to } t \approx 38\ \text{s}}$, again from $\boxed{t \approx 65\ \text{s to } t \approx 83\ \text{s}}$, and again from $\boxed{t = 90\ \text{s to } t \approx 108\ \text{s}}$.

(*d*) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t \approx 65\ \text{s to } t \approx 83\ \text{s}}$.

Slightly different answers may be obtained since the data comes from reading the graph.

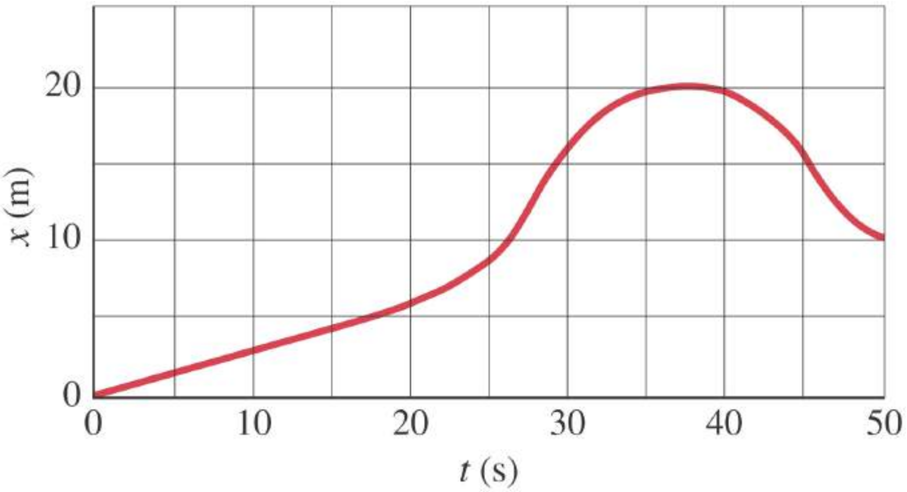
(*a*) To estimate the distance the object traveled during the first minute, we need to find the area under the graph, from $t = 0$ s to $t = 60$ s. Each "block" of the graph represents an "area" of $\Delta x = (10\ \text{m/s})(10\ \text{s}) = 100\ \text{m}$. By counting and estimating, there are about 17.5 blocks under the 1st minute of the graph, and so the distance traveled during the 1st minute is about $\boxed{1750\ \text{m}}$.

(*b*) For the second minute, there are about 5 blocks under the graph, and so the distance traveled during the second minute is about $\boxed{500\ \text{m}}$.

Alternatively, average accelerations can be estimated for various portions of the graph, and then the uniform acceleration equations may be applied. For instance, for part (*a*), break the motion up into

2. The position of a rabbit along a straight tunnel as a function of time is plotted below. What is the instantaneous velocity
    a. at t = 10.0s and,
    b. at t = 30.0s?

[2 marks]



What is the average velocity
    c. between t = 0 and t = 5.0s,
    d. between t = 25.0s and t = 30.0s and,
    e. between t = 40.0s and t = 50.0s?

[3 marks]

Slightly different answers may be obtained since the data comes from reading the graph.

(a) The instantaneous velocity is given by the slope of the tangent line to the curve. At $t = 10.0\text{ s}$, the slope is approximately $v(10) \approx \dfrac{3\,\text{m} - 0}{10.0\text{ s} - 0} = \boxed{0.3\,\text{m/s}}$.

(b) At $t = 30.0\text{ s}$, the slope of the tangent line to the curve, and thus the instantaneous velocity, is approximately $v(30) \approx \dfrac{22\text{ m} - 8\text{ m}}{35\text{ s} - 25\text{ s}} = \boxed{1.4\,\text{m/s}}$.

(c) The average velocity is given by $\overline{v} = \dfrac{x(5)\,\text{m} - x(0)\,\text{m}}{5.0\text{ s} - 0\text{ s}} = \dfrac{1.5\text{ m} - 0}{5.0\text{ s}} = \boxed{.30\,\text{m/s}}$.

(d) The average velocity is given by $\overline{v} = \dfrac{x(30)\,\text{m} - x(25)\,\text{m}}{30.0\text{ s} - 25.0\text{ s}} = \dfrac{16\text{ m} - 9\text{ m}}{5.0\text{ s}} = \boxed{1.4\,\text{m/s}}$.

(e) The average velocity is given by $\overline{v} = \dfrac{x(50)\,\text{m} - x(40)\,\text{m}}{50.0\text{ s} - 40.0\text{ s}} = \dfrac{10\text{ m} - 19.5\text{ m}}{10.0\text{ s}} = \boxed{-0.95\,\text{m/s}}$.

3. A certain type of automobile can accelerate approximately as shown in the velocity – time graph as shown below. (The short flat spots in the curve represent shifting of the gears.)

Estimate the average acceleration when it is in

a. first,
b. third,
c. fifth gear.
d. What is its average acceleration through the first four gears?

[4 marks]

Slightly different answers may be obtained since the data comes from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being "in" a certain gear.

(a) The average acceleration in first gear is given by $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{14\,\mathrm{m/s} - 0\,\mathrm{m/s}}{4\text{ s} - 0\text{ s}} = \boxed{4\,\mathrm{m/s^2}}$.

(b) The average acceleration in third gear is given by $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{37\,\mathrm{m/s} - 24\,\mathrm{m/s}}{14\text{ s} - 9\text{ s}} = \boxed{3\,\mathrm{m/s^2}}$.

(c) The average acceleration in fifth gear is given by $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{52\,\mathrm{m/s} - 44\,\mathrm{m/s}}{50\text{ s} - 27\text{ s}} = \boxed{0.35\,\mathrm{m/s^2}}$.

(d) The average acceleration through the first four gears is given by

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{44\,\mathrm{m/s} - 0\,\mathrm{m/s}}{27\text{ s} - 0\text{ s}} = \boxed{1.6\,\mathrm{m/s^2}}.$$

Topic 2.1c Kinematics Problems

Calculation-based Questions

1. A car accelerates from 13m/s to 25m/s in 6.0s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

[2 marks]

By definition, the acceleration is $a = \frac{v - v_0}{t} = \frac{25\,\text{m/s} - 13\,\text{m/s}}{6.0\ \text{s}} = \boxed{2.0\,\text{m/s}^2}$.

The distance of travel can be found from Eq. 2-11b.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 = (13\,\text{m/s})(6.0\ \text{s}) + \tfrac{1}{2}(2.0\,\text{m/s}^2)(6.0\ \text{s})^2 = \boxed{114\ \text{m}}$$

2. A car slows down from 23m/s to rest in a distance of 85m. What was the acceleration, assumed constant?

[1 mark]

The acceleration can be found from Eq. (2-11c).

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (23\,\text{m/s})^2}{2(85\ \text{m})} = \boxed{-3.1\,\text{m/s}^2}.$$

3. Estimate how long it took King Kong to fall straight down from the top of the Empire State Building (380m high) and his velocity just before he touched the ground. Ignore air resistance.

[2 marks]

Choose downward to be the positive direction, and take $y_0 = 0$ to be at the top of the Empire State Building. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\text{m/s}^2$.

(*a*) The elapsed time can be found from Eq. 2-11b, with *x* replaced by *y*.

$$y - y_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(380\ \text{m})}{9.80\,\text{m/s}^2}} = 8.806\,\text{s} \approx \boxed{8.8\ \text{s}}.$$

(*b*) The final velocity can be found from equation (2-11a).

$$v = v_0 + at = 0 + (9.80\,\text{m/s}^2)(8.806\ \text{s}) = \boxed{86\,\text{m/s}}$$

4. A stone is thrown vertically upward with a speed of 18.0m/s. How fast is it moving when it reaches a height of 11.0m and how long is required to reach this height? Why are there two answers?

[3 marks]

Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 18.0\,\mathrm{m/s}$, $a = -9.80\,\mathrm{m/s^2}$, and $y - y_0 = 11.0\ \mathrm{m}$.

(a) The velocity can be found from Eq, 2-11c, with x replaced by y.

$$v^2 = v_0^2 + 2a(y - y_0) = 0 \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2ay} = \pm\sqrt{(18.0\,\mathrm{m/s})^2 + 2(-9.80\,\mathrm{m/s^2})(11.0\ \mathrm{m})} = \pm 10.4\,\mathrm{m/s}$$

Thus the speed is $\boxed{|v| = 10.4\,\mathrm{m/s}}$

(b) The time to reach that height can be found from equation (2-11b).

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2(18.0\,\mathrm{m/s})}{-9.80\,\mathrm{m/s^2}}t + \frac{2(-11.0\ \mathrm{m})}{-9.80\,\mathrm{m/s^2}} = 0 \quad \rightarrow$$

$$t^2 - 3.673t + 2.245 = 0 \quad \rightarrow \quad \boxed{t = 2.90\ \mathrm{s}\ ,\ 0.775\ \mathrm{s}}$$

(c) $\boxed{\text{There are two times at which the object reaches that height}}$ – once on the way up $(t = 0.775\ \mathrm{s})$, and once on the way down $(t = 2.90\ \mathrm{s})$.

5. A stone is thrown vertically upward with a speed of 12.0m/s from the edge of a cliff 70.0m high. How long does it take to reach the bottom of the cliff and what is its speed before hitting? What was the total distance that it traveled? Ignore air resistance.

[3 marks]

Choose downward to be the positive direction, and $y_0 = 0$ to be at the top of the cliff. The initial velocity is $v_0 = -12.0\,\mathrm{m/s}$, the acceleration is $a = 9.80\,\mathrm{m/s^2}$, and the final location is $y = 70.0\ \mathrm{m}$.

(a) Using Eq. 2-11b and substituting y for x, we have

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow \left(4.9\,\mathrm{m/s^2}\right)t^2 - (12.0\,\mathrm{m/s})t - 70\ \mathrm{m} = 0 \quad \rightarrow \quad t = -2.749\ \mathrm{s}\ ,\ 5.198\ \mathrm{s}\ .$$

The positive answer is the physical answer: $\boxed{t = 5.20\ \mathrm{s}}$.

(b) Using Eq. 2-11a, we have $v = v_0 + at = -12.0\,\mathrm{m/s} + \left(9.80\,\mathrm{m/s^2}\right)(5.198\ \mathrm{s}) = \boxed{38.9\,\mathrm{m/s}}$.

(c) The total distance traveled will be the distance up plus the distance down. The distance down will be 70 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Then using Eq. 2-11c:

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (-12.0\,\mathrm{m/s})^2}{2\left(9.80\,\mathrm{m/s^2}\right)} = -7.35\ \mathrm{m}\ .$$

Thus the distance up is 7.35 m, the distance down is 77.35 m, and the total distance traveled is $\boxed{84.7\ \mathrm{m}}$.

Topic 2.1d Projectile Motion Problems

Conceptual Questions

(These questions are not in an IB style but instead designed to check your understanding of the concept of this topic. You should try your best to appropriately communicate your answer using prose)

1. Two cannon balls A and B are fired from the ground with identical initial speeds, but with θA larger than θB. (a) Which cannonball reaches a higher elevation? (b) which stays longer in the air? (c) Which travels farther?

(*a*) Cannonball A, with the larger angle, will reach a higher elevation. It has a larger initial vertical velocity, and so by Eq. 2-11c, will rise higher before the vertical component of velocity is 0.
(*b*) Cannonball A, with the larger angle, will stay in the air longer. It has a larger initial vertical velocity, and so takes more time to decelerate to 0 and start to fall.
(*c*) The cannonball with a launch angle closest to 45° will travel the farthest. The range is a maximum for a launch angle of 45°, and decreases for angles either larger or smaller than 45°.

2. A projectile is launched at an angle of 30° to the horizontal with a speed of 30m/s. How does the horizontal component of its velocity 1.0s after launch compare with its horizontal component of velocity 2.0s after launch?

The horizontal component of the velocity stays constant in projectile motion, assuming that air resistance is negligible. Thus the horizontal component of velocity 1.0 seconds after launch will be the same as the horizontal component of velocity 2.0 seconds after launch. In both cases the horizontal velocity will be given by $v_x = v_0 \cos\theta = (30\,\mathrm{m/s})(\cos 30^\circ) = 26\,\mathrm{m/s}$.

Calculation Based

3. A tiger leaps horizontally from a 6.5m high rock with a speed of 3.5m/s. How far from the base of the rock will she land?

Choose downward to be the positive y direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.5\,\mathrm{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\mathrm{m/s^2}$, $y_0 = 0$, and the final location $y = 6.5\,\mathrm{m}$. The time for the tiger to reach the ground is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 6.5\mathrm{m} = 0 + 0 + \tfrac{1}{2}(9.8\,\mathrm{m/s^2})t^2 \quad\rightarrow\quad t = \sqrt{\frac{2(6.5\mathrm{m})}{9.8\,\mathrm{m/s^2}}} = 1.15\ \mathrm{sec}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = (3.5\,\mathrm{m/s})(1.15\ \mathrm{sec}) = \boxed{4.0\ \mathrm{m}}$$

4. A diver running 1.8m/s dives out horizontally from the edge of a vertical cliff and 3.0s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

Choose downward to be the positive y direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $v_{x0} = 1.8\,\mathrm{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\mathrm{m/s^2}$, $y_0 = 0$, and the time of flight is $t = 3.0\ \mathrm{s}$. The height of the cliff is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad y = 0 + 0 + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(3.0\,\mathrm{s})^2 = \boxed{44\ \mathrm{m}}$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t = (1.8\,\mathrm{m/s})(3\ \mathrm{s}) = \boxed{5.4\ \mathrm{m}}$$

5. A football is kicked at ground level with a speed of 18.0m/s at an angle of 35.0° to the horizontal. How much later does it hit the ground?

Choose the point at which the football is kicked the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $v_{y0} = (18.0 \sin 35.0^\circ)\,\mathrm{m/s}$, $a_y = -9.80\,\mathrm{m/s^2}$ and the final y velocity will be the opposite of the starting y velocity (reference problem 3-28). Use Eq. 2-11a to find the time of flight.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{(-18.0 \sin 35.0^\circ)\,\mathrm{m/s} - (18.0 \sin 35.0^\circ)\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = \boxed{2.11\ \mathrm{s}}$$

6. A projectile is fired with an initial speed of 65.2m/s at an angle of 34.5° above the horizontal on a long flat firing range. Determine (a) the maximum height reached by the projectile, (b) the total time in the air, (c) the total horizontal distance covered (that is the range), and (d) the velocity of the projectile 1.50s after firing.

Choose the origin to be where the projectile is launched, and upwards to be the positive y direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin \theta_0$.

(*a*) The maximum height is found from Eq. 2-11c, $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$, with $v_y = 0$ at the maximum height.

$$y_{\max} = 0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{-v_0^2 \sin^2 \theta_0}{-2g} = \frac{v_0^2 \sin^2 \theta_0}{2g} = \frac{(65.2\,\mathrm{m/s})^2 \sin^2 34.5^\circ}{2(9.80\,\mathrm{m/s^2})} = \boxed{69.6\ \mathrm{m}}$$

(*b*) The total time in the air is found from Eq. 2-11b, with a total vertical displacement of 0 for the ball to reach the ground.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = v_0 \sin \theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{2v_0 \sin \theta_0}{g} = \frac{2(65.2\,\mathrm{m/s}) \sin 34.5^\circ}{(9.80\,\mathrm{m/s^2})} = \boxed{7.54\ \mathrm{s}} \text{ and } t = 0$$

The time of 0 represents the launching of the ball.

(*c*) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0 \cos \theta_0)t = (65.2\,\mathrm{m/s})(\cos 34.5^\circ)(7.54\ \mathrm{s}) = \boxed{405\ \mathrm{m}}$$

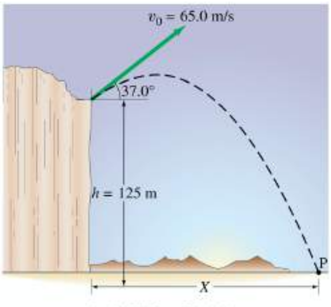
(*d*) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_0 \cos \theta_0 = (65.2\,\mathrm{m/s})(\cos 34.5^\circ) = 53.7\,\mathrm{m/s}$. The vertical velocity is found from Eq. 2-11a.

$$v_y = v_{y0} + at = v_0 \sin \theta_0 - gt = (65.2\,\mathrm{m/s}) \sin 34.5^\circ - (9.80\,\mathrm{m/s^2})(1.50\,\mathrm{s}) = 22.2\,\mathrm{m/s}$$

Thus the speed of the projectile is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{53.7^2 + 22.2^2} = \boxed{58.1\,\mathrm{m/s}}$.

The direction above the horizontal is given by $\theta = \tan^{-1} \dfrac{v_y}{v_x} = \tan^{-1} \dfrac{22.2}{53.7} = \boxed{22.5^\circ}$.

7. A projectile is shot from the edge of a cliff 125m above ground level with an initial speed of 65.0m/s at an angle of 37.0° with the horizontal, as shown below. (a) Determine the time taken by the projectile to hit point P at ground level. (b) Determine the range X of the projectile as measured from the base of the cliff. At the instant just before the projectile hits point P, find (c) the horizontal and the vertical components of its velocity, (d) the magnitude of the velocity, and (e) the angle made by the velocity vector with the horizontal. (f) Find the maximum height above the cliff top reached by the projectile.



Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive y direction. For the projectile, $v_0 = 65.0\,\mathrm{m/s}$, $\theta_0 = 37.0^\circ$, $a_y = -g$, $y_0 = 125$, and $v_{y0} = v_0 \sin\theta_0$

(a) The time taken to reach the ground is found from Eq. 2-11b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = 125 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t = \frac{-v_0 \sin\theta_0 \pm \sqrt{v_0^2 \sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)(125)}}{2\left(-\tfrac{1}{2}g\right)} = \frac{-39.1 \pm 63.1}{-9.8} = 10.4\ \mathrm{s}\,,\ -2.45\ \mathrm{s} = \boxed{10.4\ \mathrm{s}}$$

Choose the positive sign since the projectile was launched at time $t = 0$.

(b) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0 \cos\theta_0)t = (65.0\,\mathrm{m/s})\cos 37.0^\circ\,(10.4\ \mathrm{s}) = \boxed{541\ \mathrm{m}}$$

(c) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0\cos\theta_0 = (65.0\,\mathrm{m/s})\cos 37.0^\circ = \boxed{51.9\,\mathrm{m/s}}$. The vertical component is found from Eq. 2-11a.

$$v_y = v_{y0} + at = v_0\sin\theta_0 - gt = (65.0\,\mathrm{m/s})\sin 37.0^\circ - (9.80\,\mathrm{m/s^2})(10.4\ \mathrm{s})$$

$$= \boxed{-63.1\,\mathrm{m/s}}$$

(d) The magnitude of the velocity is found from the x and y components calculated in part c) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(51.9\,\mathrm{m/s})^2 + (-63.1\,\mathrm{m/s})^2} = \boxed{81.7\,\mathrm{m/s}}$$

(e) The direction of the velocity is $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{-63.1}{51.9} = -50.6^\circ$, and so the object is moving $\boxed{50.6^\circ \text{ below the horizon}}$.

(f) The maximum height above the cliff top reached by the projectile will occur when the y-velocity is 0, and is found from Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \quad\rightarrow\quad 0 = v_0^2\sin^2\theta_0 - 2gy_{\max}$$

$$y_{\max} = \frac{v_0^2\sin^2\theta_0}{2g} = \frac{(65.0\,\mathrm{m/s})^2\sin^2 37.0^\circ}{2(9.80\,\mathrm{m/s^2})} = \boxed{78.1\ \mathrm{m}}$$

Topic 2.2a Kinematics Problems

Calculation-based Questions

1. A net force of 265N accelerates a bike and rider at 2.30m/s^2. What is the mass of the bike and the rider together?

[1 mark]

Use Newton's second law to calculate the mass.

$$\sum F = ma \quad \rightarrow \quad m = \frac{\sum F}{a} = \frac{265\ \text{N}}{2.30\ \text{m/s}^2} = \boxed{115\ \text{kg}}$$

2. What is the weight of a 76kg astronaut
   a. on Earth
   b. on the Moon (g = 1.7m/s^2)
   c. on Mars (g = 3.7m/s^2)
   d. in outer space traveling with constant velocity?

[4 marks]

In all cases, $W = mg$, where g changes with location.

$$(a)\quad W_{\text{Earth}} = mg_{\text{Earth}} = (76\ \text{kg})(9.8\ \text{m/s}^2) = \boxed{7.4\times 10^2\ \text{N}}$$

$$(b)\quad W_{\text{Moon}} = mg_{\text{Moon}} = (76\ \text{kg})(1.7\ \text{m/s}^2) = \boxed{1.3\times 10^2\ \text{N}}$$

$$(c)\quad W_{\text{Mars}} = mg_{\text{Mars}} = (76\ \text{kg})(3.7\ \text{m/s}^2) = \boxed{2.8\times 10^2\ \text{N}}$$

$$(d)\quad W_{\text{Space}} = mg_{\text{Space}} = (76\ \text{kg})(0\ \text{m/s}^2) = \boxed{0\ \text{N}}$$

3. What is the average force required to stop a 1100-kg car in 8.0s if the car is travelling at 95km/h?

[3 marks]

Find the average acceleration from Eq. 2-2. The average force on the car is found from Newton's second law.

$$v = 0 \qquad v_0 = (95\ \text{km/h})\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right) = 26.4\ \text{m/s} \qquad a_{avg} = \frac{v - v_0}{t} = \frac{0 - 26.4\ \text{m/s}}{8.0\ \text{s}} = -3.30\ \text{m/s}^2$$

$$F_{avg} = ma_{avg} = (1100\ \text{kg})(-3.3\ \text{m/s}^2) = \boxed{-3.6\times 10^3\ \text{N}}$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity.

4. What is the average force needed to accelerate a 7.00g pellet from rest to 125m/s over a distance of 0.800m along the barrel of the rifle?

[2 marks]

The average force on the pellet is its mass times its average acceleration. The average acceleration is found from Eq. 2-11c. For the pellet, $v_0 = 0$, $v = 125\ \text{m/s}$, and $x - x_0 = 0.800\ \text{m}$.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(125\ \text{m/s})^2 - 0}{2(0.800\ \text{m})} = 9770\ \text{m/s}^2$$

$$F_{avg} = ma_{avg} = (7.00\times 10^{-3}\ \text{kg})(9770\ \text{m/s}^2) = \boxed{68.4\ \text{N}}$$

5. A 0.140kg baseball traveling at 35.0m/s strikes the catcher's mit, which, in bringing the ball to rest, recoils backward 11.0cm. What was the average force applied by the ball on the glove?

[2 marks]

The problem asks for the average force on the glove, which in a direct calculation would require knowledge about the mass of the glove and the acceleration of the glove. But no information about the glove is given. By Newton's 3rd law, the force exerted by the ball on the glove is equal and opposite to the force exerted by the glove on the ball. So calculate the average force on the ball, and then take the opposite of that result to find the average force on the glove. The average force on the ball is its mass times its average acceleration. Use Eq. 2-11c to find the acceleration of the ball, with $v = 0$, $v_0 = 35.0\,\mathrm{m/s}$, and $x - x_0 = 0.110\ \mathrm{m}$. The initial direction of the ball is the positive direction.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (35.0\,\mathrm{m/s})^2}{2(0.110\ \mathrm{m})} = -5568\ \mathrm{m/s^2}$$

$$F_{avg} = ma_{avg} = (0.140\ \mathrm{kg})(-5568\ \mathrm{m/s^2}) = \boxed{-7.80 \times 10^2\,\mathrm{N}}$$

Thus the average force on the glove was 780 N, in the direction of the initial velocity of the ball.

Topic 2.2b Kinematics Problems

Conceptual Questions

(These questions are not in an IB style but instead designed to check your understanding of the concept of this topic. You should try your best to appropriately communicate your answer using prose)

1. Why does a child, sat in a toy wagon, seem to fall backward when you give the wagon a sharp pull forward?

   The child has inertia – the property of an object to resist change. As you pull the wagon forward, only the feet of the child are pulled forward and the rest of the body wants to stay in place. Hence, the child seem to fall backward.

2. If the acceleration of an object is zero, are no forces acting on it? If only one force acts on the object, can the object have zero acceleration? Can it have zero velocity? Explain.

   The net force has to be zero if acceleration is zero. This does not mean that there are no forces acting on it, just that all forces acting on the object are balanced (cancel each other out). If only one force acts on the objects, this is the net force and the object accelerates.
   The velocity can be zero even if the net force (and hence, the acceleration) is not zero. However, this velocity has to change in the next instant.

3. If you walk along a log floating on a lake, why does the log move in the opposite direction?

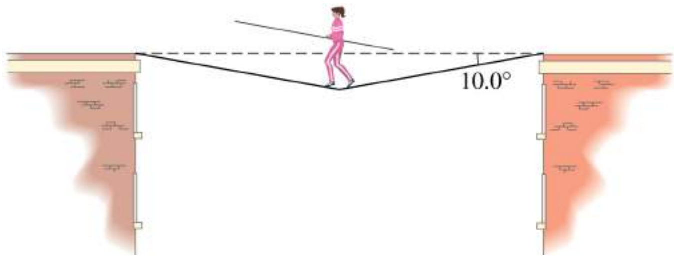
   The log is pushing you forward, but according to N3 you are also pushing the log backwards (in the opposite direction)

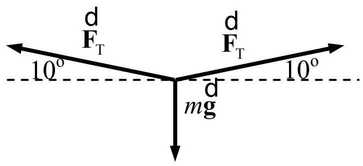
Topic 2.2c Kinematics Problems

Calculation-based Questions

1. Arlene is to walk across a "high-wire" strung horizontally between two buildings 10.0m apart. The sag in the rope when she is at the mid-point is 10.0° as shown. If her mass is 50.0kg, what is the tension in the rope at this point?

[2 marks]





Consider the point in the rope directly below Arlene. That point can be analyzed as having three forces on it – Arlene's weight, the tension in the rope towards the right point of connection, and the tension in the rope towards the left point of connection. Assuming the rope is massless, those two tensions will be of the same magnitude. Since the point is not accelerating the sum of the forces must be zero. In particular, consider the sum of the vertical forces on that point, with UP as the positive direction.

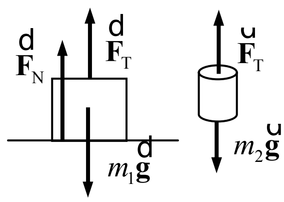
$$\sum F = F_T \sin 10.0^\circ + F_T \sin 10.0^\circ - mg = 0 \rightarrow$$

$$F_T = \frac{mg}{2\sin 10.0^\circ} = \frac{(50.0\ \text{kg})(9.80\ \text{m/s}^2)}{2\sin 10.0^\circ} = \boxed{1.41 \times 10^3\ \text{N}}$$

2. A box weighing 77.0N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end. Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs
   a. 30.0N
   b. 60.0N
   c. 90.0N

[4 marks]

(Hint: You should sketch a free-body diagram for the general case)



Free body diagrams for the box and the weight are shown below. The tension exerts the same magnitude of force on both objects.

(*a*) If the weight of the hanging weight is less than the weight of the box, the objects will not move, and the tension will be the same as the weight of the hanging weight. The acceleration of the box will also be zero, and so the sum of the forces on it will be zero. For the box,

$$F_N + F_T - m_1 g = 0 \rightarrow \quad F_N = m_1 g - F_T = m_1 g - m_2 g = 77.0\text{N} - 30.0\ \text{N} = \boxed{47.0\ \text{N}}$$

(*b*) The same analysis as for part (*a*) applies here.

$$F_N = m_1 g - m_2 g = 77.0\ \text{N} - 60.0\ \text{N} = \boxed{17.0\ \text{N}}$$

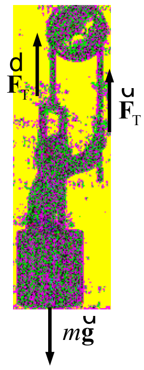
(*c*) Since the hanging weight has more weight than the box on the table, the box on the table will be lifted up off the table, and normal force of the table on the box will be $\boxed{0\ \text{N}}$.

3. A window washer pulls herself upward using a bucket-pulley system as shown.
    a. Sketch a free-body diagram showing the force of gravity and the force exerted by the rope (tension).
    b. How hard must she pull downward to raise herself slowly at constant speed?
    c. If she increases this force by 15%, what will her acceleration by?

    Assume the mass of the person and the bucket is 65kg.

[4 marks]

The window washer pulls down on the rope with her hands with a tension force F_T, so the rope pulls up on her hands with a tension force F_T. The tension in the rope is also applied at the other end of the rope, where it attaches to the bucket. Thus there is another force F_T pulling up on the bucket. The bucket-washer combination thus has a net force of $2F_T$ upwards. See the adjacent free-body diagram, showing only forces on the bucket-washer combination, not forces exerted by the combination (the pull down on the rope by the person) or internal forces (normal force of bucket on person).



(*a*) Write Newton's 2nd law in the vertical direction, with up as positive. The net force must be zero if the bucket and washer have a constant speed.

$$\sum F = F_T + F_T - mg = 0 \rightarrow \quad 2F_T = mg \rightarrow$$

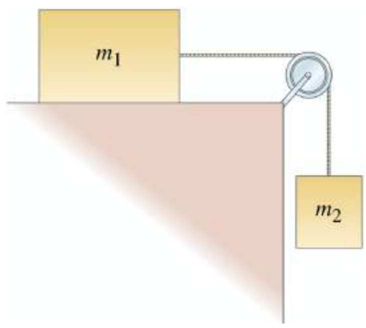
$$F_T = \frac{mg}{2} = \frac{(65\ \text{kg})(9.8\ \text{m/s}^2)}{2} = \boxed{320\ \text{N}}$$

(*b*) Now the force is increased by 15%, so $F_T = 320\ \text{N}(1.15) = 368\ \text{N}$. Again write Newton's 2nd law, but with a non-zero acceleration.

$$\sum F = F_T + F_T - mg = ma \rightarrow$$

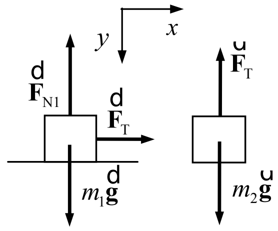
$$a = \frac{2F_T - mg}{m} = \frac{2(368\ \text{N}) - (65\ \text{kg})(9.80\ \text{m/s}^2)}{65\ \text{kg}} = \boxed{1.5\ \text{m/s}^2}$$

4. The diagram below shows a block (mass m_A) on a smooth horizontal surface, connected by a thin cord that passes over a pulley to a second block (m_B), which hangs vertically.



    a. Draw a free-body diagram for each block, showing the force of gravity on each, the force (tension) exerted by the cord, and any normal force.
    b. Apply Newton's second law to find formulas for the acceleration of the system and for the tension in the cord. Ignore friction and the masses of the pulley and cord.

[3 marks]



(a) See the free-body diagrams included.

(b) For block 1, since there is no motion in the vertical direction, we have $F_{N1} = m_1 g$. We write Newton's 2nd law for the x direction: $\sum F_{1x} = F_T = m_1 a_{1x}$. For block 2, we only need to consider vertical forces: $\sum F_{2y} = m_2 g - F_T = m_2 a_{2y}$. Since the two blocks are connected, the magnitudes of their accelerations will be the same, and so let $a_{1x} = a_{2y} = a$. Combine the two force equations from above, and solve for a by substitution.

$$F_T = m_1 a \qquad m_2 g - F_T = m_2 a \quad \rightarrow \quad m_2 g - m_1 a = m_2 a \quad \rightarrow$$

$$m_1 a + m_2 a = m_2 g \quad \rightarrow \quad \boxed{a = g\frac{m_2}{m_1 + m_2} \qquad F_T = m_1 a = g\frac{m_1 m_2}{m_1 + m_2}}$$

Topic 2.3a Work and Energy

Conceptual Questions

(These questions are not in an IB style but instead designed to check your understanding of the concept of this topic. You should try your best to appropriately communicate your answer using prose)

1. You have two springs that are identical except that spring 1 is stiffer than spring 2 ($k_1 > k_2$). On which spring is more work done if (a) they are stretched using the same force, (b) stretched the same distance?

   (*a*) In this case, the same force is applied to both springs. Spring 1 will stretch less, and so more work is done on spring 2.

   (*b*) In this case, both springs are stretched the same distance. It takes more force to stretch spring 1, and so more work is done on spring 1.

2. When a rubber bouncy-ball is dropped, can it rebound to a height greater than its original height? Explain.

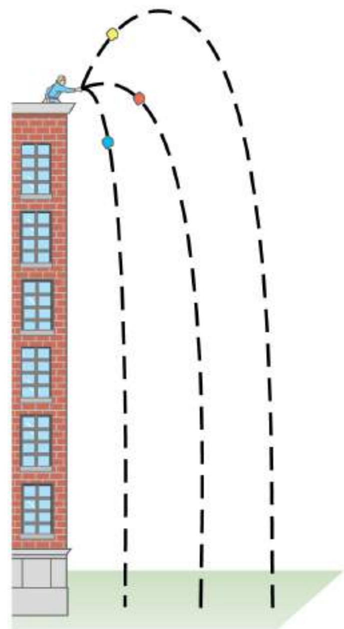
   The superball cannot rebound to a height greater than its original height when dropped. If it did, it would violate conservation of energy. When a ball collides with the floor, the KE of the ball is converted into elastic PE by deforming the ball, much like compressing a spring. Then as the ball springs back to its original shape, that elastic PE is converted to back to KE. But that process is "lossy" – not all of the elastic PE gets converted back to KE. Some of the PE is lost, primarily to friction. The superball rebounds higher than many other balls because it is less "lossy" in its rebound than many other materials.

3. Why is it easier to climb a mountain via a zigzag trail than to climb straight up?

   The climber does the same amount of work whether climbing straight up or via a zig-zag path, ignoring dissipative forces. But if a longer zig-zag path is taken, it takes more time to do the work, and so the power output needed from the climber is less. That will make the climb easier. It is easier for the human body to generate a small amount of power for long periods of time rather than to generate a large power for a small period of time.

4. Water balloons are tossed from the roof of a building, all with the same speed but with different launch angles. Which one has the highest speed on impact? Ignore air resistance.

   Since each balloon has the same initial kinetic energy, and each balloon undergoes the same overall change in gravitational PE, each balloon will have the same kinetic energy at the ground, and so each one has the same speed at impact.



Topic 2.3b Work done by a constant force

Calculation-based Questions

1. How much work is done by the gravitational force when a 256kg pile driver falls 2.80m?

[1 mark]

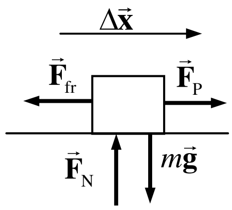
The force and the displacement are both downwards, so the angle between them is 0°.

$$W_{\mathrm{G}} = mgd\cos\theta = (265\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(2.80\ \mathrm{m})\cos 0^{\circ} = \boxed{7.27\times10^{3}\ \mathrm{J}}$$

2. A 1300-N crate rests on the floor. How much work is required to move it at a constant speed…
   a. 4.0m along the floor against a friction force of 230N, and
   b. 4.0m vertically?

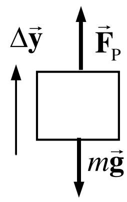
[2 mark]

(*a*) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_{\mathrm{P}} = F_{\mathrm{fr}} = 230\ \mathrm{N}$. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is 0°.



$$W_{\mathrm{P}} = F_{\mathrm{P}}d\cos 0^{\circ} = (230\ \mathrm{N})(4.0\ \mathrm{m})(1) = \boxed{9.2\times10^{2}\ \mathrm{J}}$$

(*b*) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0°.



$$W_{\mathrm{P}} = F_{\mathrm{P}}d\cos 0^{\circ} = mgd = (1300\ \mathrm{N})(4.0\ \mathrm{m}) = \boxed{5.2\times10^{3}\ \mathrm{J}}$$

3. A box of mass 5.0kg is accelerated from rest across a floor at a rate of 2.0m/s2 for 7.0s. Find the net work done on the box.

[2 marks]

Since the acceleration of the box is constant, use Eq. 2-11b to find the distance moved. Assume that the box starts from rest.

$$\Delta x = x - x_0 = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(2.0\ \mathrm{m/s^2})(7\ \mathrm{s})^2 = 49\ \mathrm{m}$$

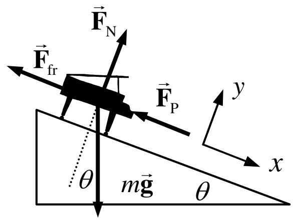
Then the work done in moving the crate is

$$W = F\Delta x\cos 0^{\circ} = ma\Delta x = (5\ \mathrm{kg})(2.0\ \mathrm{m/s^2})(49\ \mathrm{m}) = \boxed{4.9\times10^{2}\ \mathrm{J}}$$

4. A 330-kg piano slides 3.6m down a 28° incline and is kept from accelerating by a man who is pushing back on it parallel to the incline. The effective coefficient of friction μ is 0.40 and the force due to friction can be calculated by F = μN where is the normal force.
   Calculate:
   a. the force exerted by the man.
   b. the work done by the man on the piano.
   c. the work done by the friction force.
   d. the work done by the force of gravity.
   e. the net work done on the piano.

[10 marks]

(Hint: Draw the free-body diagram. You will need to resolve forces parallel and perpendicular to the incline.)



The piano is moving with a constant velocity down the plane. $\vec{\mathbf{F}}_P$ is the force of the man pushing on the piano.

(*a*) Write Newton's 2nd law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_P - F_{fr} = 0 \quad \rightarrow$$

$$F_P = mg\sin\theta - F_{fr} = mg\left(\sin\theta - \mu_k\cos\theta\right)$$

$$= (330\ \text{kg})\left(9.80\ \text{m/s}^2\right)\left(\sin 28^\circ - 0.40\cos 28^\circ\right) = \boxed{3.8\times10^2\ \text{N}}$$

(*b*) The work done by the man is the work done by $\vec{\mathbf{F}}_P$. The angle between $\vec{\mathbf{F}}_P$ and the direction of motion is 180°.

$$W_P = F_P d\cos 180^\circ = -(380\ \text{N})(3.6\ \text{m}) = \boxed{-1.4\times10^3\ \text{J}}.$$

(*c*) The angle between $\vec{\mathbf{F}}_{fr}$ and the direction of motion is 180°.

$$W_{fr} = F_{fr} d\cos 180^\circ = -\mu_k mgd\cos\theta = -(0.40)(330\ \text{kg})\left(9.8\ \text{m/s}^2\right)(3.6\ \text{m})\cos 28^\circ$$

$$= \boxed{-4.1\times10^3\ \text{J}}$$

(*d*) The angle between the force of gravity and the direction of motion is 62°. So the work done by gravity is

$$W_G = F_G d\cos 62^\circ = mgd\cos 62^\circ = (330\ \text{kg})\left(9.8\ \text{m/s}^2\right)(3.6\ \text{m})\cos 62^\circ = \boxed{5.5\times10^3\ \text{J}}.$$

(*e*) Since the piano is unaccelerated, the net force on the piano is 0, and so the net work done on the piano is also 0. This can also be seen by adding the three work amounts calculated.

$$W_{Net} = W_P + W_{fr} + W_G = -1400\ \text{J} - 4100\ \text{J} + 5500\ \text{J} = \boxed{0\ \text{J}}$$

Topic 2.3c Mechanical Energy

Calculation-based Questions

1. If the KE of an arrow is doubled, by what factor has its speed increased? If its speed is doubled, by what factor does its KE increase?

[2 marks]

(a) Since $KE = \frac{1}{2}mv^2$, then $v = \sqrt{2(KE)/m}$ and so $v \propto \sqrt{KE}$. Thus if the kinetic energy is doubled, the speed will be multiplied by a factor of $\boxed{\sqrt{2}}$.

(b) Since $KE = \frac{1}{2}mv^2$, then $KE \propto v^2$. Thus if the speed is doubled, the kinetic energy will be multiplied by a factor of $\boxed{4}$.

2. How much work must be done to stop a 1250-kg car travelling at 105km/h?

[2 marks]

The work done on the car is equal to the change in its kinetic energy, and so

$$W = KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(1250\,\text{kg})\left[(105\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = \boxed{-5.32\times10^5\,\text{J}}$$

3. A spring has a stiffness constant of, k, of 440N/m. How much must this spring be stretched to store 25J of potential energy?

[1 mark]

The elastic PE is given by $PE_{\text{elastic}} = \frac{1}{2}kx^2$ where x is the distance of stretching or compressing of the spring from its natural length.

$$x = \sqrt{\frac{2PE_{\text{elastic}}}{k}} = \sqrt{\frac{2(25\text{ J})}{440\,\text{N/m}}} = \boxed{0.34\text{ m}}$$

4. By how much does the gravitational potential energy of a 64kg pole-vaulter change if his centre of mass rises about 4.0m during the jump?

[1 mark]

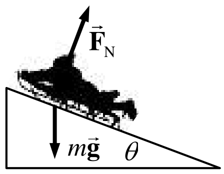
Subtract the initial gravitational PE from the final gravitational PE.

$$\Delta PE_{\text{grav}} = mgy_2 - mgy_1 = mg(y_2 - y_1) = (64\text{ kg})(9.8\,\text{m/s}^2)(4.0\text{ m}) = \boxed{2.5\times10^3\,\text{J}}$$

5. A sled is initially given a shove up a frictionless 28.0° incline. It reaches a maximum vertical height of 1.35m higher than where it started. What was the initial speed?

[2 marks]

The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for PE $(y=0)$. We have $y_1=0$, $v_2=0$, and $y_2=1.35\text{ m}$. Solve for v_1, the speed at the bottom.



$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2+0=0+mgy_2 \;\rightarrow$$

$$v_1=\sqrt{2gy_2}=\sqrt{2\left(9.80\text{ m/s}^2\right)\left(1.35\text{ m}\right)}=\boxed{5.14\text{ m/s}}$$

Notice that the angle is not used in the calculation.

6. Two railroad cars, each of mass 7650kg and traveling 95km/h in opposite directions, collide head-on and come to rest. How much thermal energy is produced in this collision?

[2 marks]

Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

$$E_{\text{initial}}=E_{\text{final}} \;\rightarrow\; \tfrac{1}{2}mv^2=E_{\text{thermal}}=\tfrac{1}{2}(2)(7650\text{ kg})\left[(95\text{ km/h})\left(\frac{0.238\text{ m/s}}{1\text{ km/h}}\right)\right]^2=\boxed{5.3\times10^6\text{ J}}$$

7. You drop a ball from a height of 2.0m, and it bounces back to a height of 1.5m.
   a. What fraction of its initial energy is lost during the bounce?
   b. What is the ball's speed just as it leaves the ground after the bounce?
   c. Where did the energy go?

[5 marks]

(*a*) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy "lost". The energy at the two heights is all gravitational PE, since the ball has no KE at those maximum heights.

$$E_{\text{lost}}=E_{\text{initial}}-E_{\text{final}}=mgy_{\text{initial}}-mgy_{\text{final}}$$

$$\frac{E_{\text{lost}}}{E_{\text{initial}}}=\frac{mgy_{\text{initial}}-mgy_{\text{final}}}{mgy_{\text{initial}}}=\frac{y_{\text{initial}}-y_{\text{final}}}{y_{\text{initial}}}=\frac{2.0\text{ m}-1.5\text{ m}}{2.0\text{ m}}=0.25=\boxed{25\%}$$

(*b*) Due to energy conservation, the KE of the ball just as it leaves the ground is equal to its final PE.

$$PE_{\text{final}}=KE_{\text{ground}} \;\rightarrow\; mgy_{\text{final}}=\tfrac{1}{2}mv_{\text{ground}}^2 \;\rightarrow$$

$$v_{\text{ground}}=\sqrt{2gy_{\text{final}}}=\sqrt{2\left(9.8\text{ m/s}^2\right)\left(1.5\text{ m}\right)}=\boxed{5.4\text{ m/s}}$$

(*c*) The energy "lost" was changed primarily into heat energy – the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves). Some may have been lost due to non-elastic deformation of the ball or ground.

8. How long will it take a 1750-W motor to lift a 315kg piano to a sixth-story window 16.0m above?

The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W=Fd\cos 0^\circ=mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P=\frac{W}{t}=\frac{mgh}{t} \;\rightarrow\; t=\frac{mgh}{P}=\frac{(315\text{ kg})\left(9.80\text{ m/s}^2\right)(16.0\text{ m})}{1750\text{ W}}=\boxed{28.2\text{ s}}$$

9. A car generates 18hp (1hp = 746W) when travelling at a steady 88km/h, what must the average force exerted on the car due to friction and air resistance?

[2 marks]

The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is given by $P = \frac{W}{t} = \frac{Fd}{t} = F\frac{d}{t} = Fv$.

Thus the force to propel the car forward is found by $F = P/v$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus the total resistive force is also found by $F = P/v$.

$$F = \frac{P}{v} = \frac{(18\text{ hp})(746\text{ W}/1\text{ hp})}{(88\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)} = \boxed{5.5\times10^{2}\,\text{N}}$$

Topic 2.4a Momentum Problems

Conceptual Questions

(These questions are not in an IB style but instead designed to check your understanding of the concept of this topic. You should try your best to appropriately communicate your answer using prose)

1. When you release an inflated but untied balloon, why does it fly across the room?

   When you release an inflated but untied balloon at rest, the gas inside the balloon (at high pressure) rushes out the open end of the balloon. That escaping gas and the balloon form a closed system, and so the momentum of the system is conserved. The balloon and remaining gas acquires a momentum equal and opposite to the momentum of the escaping gas, and so move in the opposite direction to the escaping gas.

2. Cars used to be built as rigid as possible to withstand collisions. Today, though, cars are designed to have "crumple zones" that collapse upon impact. What is the advantage of this new design?

   "Crumple zones" are similar to air bags in that they increase the time of interaction during a collision, and therefore lower the average force required for the change in momentum that the car undergoes in the collision.

3. Is it possible for an object to receive a larger impulse from a small force than from a large force? Explain.

   The impulse is the product of the force and the time duration that the force is applied. So the impulse from a small force applied over a long time can be larger than the impulse applied by a large force over a small time.

Calculation-based Questions

4. A constant friction force of 25N acts on a 65kg skier for 20s. What is the skier's change in velocity?

   [2 marks]

   From Newton's second law, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$. For a constant mass object, $\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$. Equate the two expressions for $\Delta\vec{\mathbf{p}}$.

   $$\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}} \quad\rightarrow\quad \Delta\vec{\mathbf{v}} = \frac{\vec{\mathbf{F}}\Delta t}{m}.$$

   If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

   $$\Delta v = -\frac{F\Delta t}{m} = -\frac{(25\text{ N})(20\text{ s})}{65\text{ kg}} = \boxed{-7.7\text{ m/s}}$$

   The skier loses 7.7 m/s of speed.

5. A 0.145kg baseball pitched at 39.0m/s is hit on a horizontal line drive straight back at the pitcher at 52.0m/s. If the contact time between the bat and the ball is 3.00x10-3s, calculate the average force between the bat and the ball during contact.

   [2 marks]

   Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

   $$\Delta p = F\Delta t = m\Delta v \quad\rightarrow$$

   $$F = m\frac{\Delta v}{\Delta t} = (0.145\text{ kg})\left(\frac{52.0\text{ m/s} - -39.0\text{ m/s}}{3.00\times10^{-3}\text{ s}}\right) = \boxed{4.40\times10^{3}\text{ N, towards the pitcher}}$$

6. A child in a boat throws a 6.40kg package out horizontally with a speed of 10.0m/s. Calculate the velocity of the boat immediately after, assuming it was initially at rest. The mass of the child is 26.0kg and that of the boat is 45.0kg. Ignore water resistance.

[2 marks]

The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let "A" represent the boat and child together, and let "B" represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)v = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(6.40\text{ kg})(10.0\text{ m/s})}{(26.0\text{ kg} + 45.0\text{ kg})} = \boxed{-0.901\text{ m/s}}$$

The boat and child move in the opposite direction as the thrown package.

7. A 12,600-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0m/s. A 5350-kg load, initially at rest, is dropped onto the car. What will be the car's new speed?

[2 marks]

Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let "A" represent the car, and "B" represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = (m_A + m_B)v' \rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(12{,}600\text{ kg})(18.0\text{ m/s}) + 0}{(12{,}600\text{ kg}) + (5350\text{ kg})} = \boxed{12.6\text{ m/s}}$$

8. A 9300-kg boxcar travelling at 15.0m/s strikes a second boxcar at rest. The two stick together and move off with a common speed of 6.0m/s. What is the mass of the second car?

[2 marks]

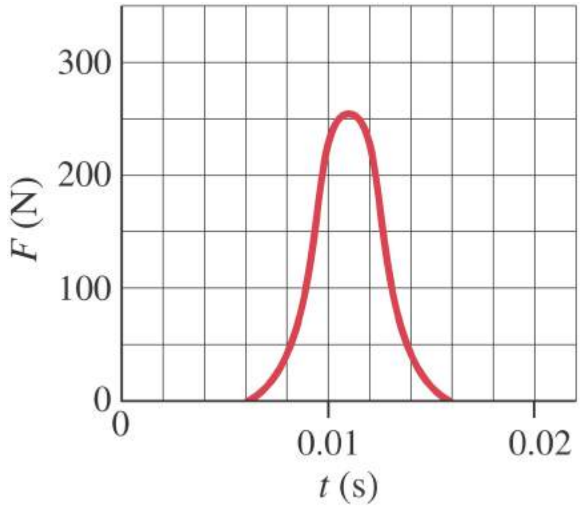
Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let "A" represent the first car, and "B" represent the second car. Momentum will be conserved in the collision. Note that $v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = (m_A + m_B)v' \rightarrow$$

$$m_B = \frac{m_A(v_A - v')}{v'} = \frac{(9300\text{ kg})(15.0\text{ m/s} - 6.0\text{ m/s})}{6.0\text{ m/s}} = \boxed{1.4\times10^4\text{ kg}}$$

9. Suppose the force acting on a tennis ball (mass 0.060kg) points in the +x direction and is given by the graph as a function of time. Use graphical methods to estimate
   a. the total impulse given the ball, and
   b. the velocity of the ball after being struck, assuming the ball is being served so it is nearly at rest initially.

[2 marks]



(a) The impulse given the ball is the area under the F vs. t graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.01 sec.

$$\Delta p = \tfrac{1}{2}(250\ \text{N})(0.01\ \text{s}) = \boxed{1.25\text{N}\bullet\text{s}}$$

(b) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p = m\Delta v = m\left(v_f - v_i\right) \quad \rightarrow \quad v_f = v_i + \frac{\Delta p}{m} = 0 + \frac{1.25\text{N}\bullet\text{s}}{6.0\times 10^{-2}\,\text{kg}} = \boxed{21\,\text{m/s}}$$

Topic 2.4b Momentum and Energy Problems

Calculation-based Questions

1. A 0.450kg ice puck, moving east with a speed of 3.00m/s, has a head on collision with a 0.900kg puck initially at rest. Assuming a perfectly elastic collision, what will be the speed and direction of each object after the collision?

[4 marks]

Let A represent the 0.450-kg puck, and let B represent the 0.900-kg puck. The initial direction of puck A is the positive direction. We have $v_A = 3.00\,\mathrm{m/s}$ and $v_B = 0$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad m_A v_A = m_A v'_A + m_B (v_A + v'_A) \quad \rightarrow$$

$$v'_A = \frac{(m_A - m_B)}{(m_A + m_B)} v_A = \frac{-0.450\ \mathrm{kg}}{1.350\ \mathrm{kg}}(3.00\,\mathrm{m/s}) = -1.00\,\mathrm{m/s} = \boxed{1.00\,\mathrm{m/s}\,(\mathrm{west})}$$

$$v'_B = v_A + v'_A = 3.00\,\mathrm{m/s} - 1.00\,\mathrm{m/s} = \boxed{2.00\,\mathrm{m/s}\,(\mathrm{east})}$$

2. Two billiard balls of equal mass undergo a perfectly elastic head-on collision. If one ball’s initial speed was 2.00m/s, and the other’s was 3.00m/s in the opposite direction, what will be their speeds after the collision?

[4 marks]

Let A represent the ball moving at 2.00 m/s, and call that direction the positive direction. Let B represent the ball moving at 3.00 m/s in the opposite direction. So $v_A = 2.00\,\mathrm{m/s}$ and $v_B = -3.00\,\mathrm{m/s}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = 5.00\,\mathrm{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision, noting that $m_A = m_B$.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad v_A + v_B = v'_A + v'_B \quad \rightarrow$$

$$-1.00\,\mathrm{m/s} = v'_A + (v'_A + 5.00\,\mathrm{m/s}) \quad \rightarrow \quad 2v'_A = -6.00\,\mathrm{m/s} \quad \rightarrow \quad v'_A = \boxed{-3.00\,\mathrm{m/s}}$$

$$v'_B = 5.00\,\mathrm{m/s} + v'_A = \boxed{2.00\,\mathrm{m/s}}$$

The two balls have exchanged velocities. This will always be true for 1-D elastic collisions of objects of equal mass.

3. A 10,000kg railroad car, A travelling at a speed of 24.0m/s strikes an identical car, B, at rest. If the cars lock together as a result of the collision, calculate:
    a. the common speed after the collision.
    b. how much of the initial kinetic energy is transformed to thermal or other forms of energy.

[4 marks]

APPROACH We choose our system to be the two railroad cars. We consider a very brief time interval, from just before the collision until just after, so that external forces such as friction can be ignored. Then we apply conservation of momentum.

SOLUTION The initial total momentum is

$$p_{\text{initial}} = m_A v_A + m_B v_B = m_A v_A$$

because car B is at rest initially ($v_B = 0$). The direction is to the right in the $+x$ direction. After the collision, the two cars become attached, so they will have the same speed, call it v'. Then the total momentum after the collision is

$$p_{\text{final}} = (m_A + m_B)v'.$$

We have assumed there are no external forces, so momentum is conserved:

$$p_{\text{initial}} = p_{\text{final}}$$

$$m_A v_A = (m_A + m_B)v'.$$

Solving for v', we obtain

$$v' = \frac{m_A}{m_A + m_B} v_A = \left(\frac{10{,}000\text{ kg}}{10{,}000\text{ kg} + 10{,}000\text{ kg}}\right)(24.0\text{ m/s}) = 12.0\text{ m/s},$$

to the right. Their mutual speed after collision is half the initial speed of car A.

NOTE We kept symbols until the very end, so we have an equation we can use in other (related) situations.

APPROACH The railroad cars stick together after the collision, so this is a completely inelastic collision. By subtracting the total kinetic energy after the collision from the total initial kinetic energy, we can find how much energy is transformed to other types of energy.

SOLUTION Before the collision, only car A is moving, so the total initial kinetic energy is

$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}(10{,}000\text{ kg})(24.0\text{ m/s})^2 = 2.88 \times 10^6\text{ J}.$$

After the collision, both cars are moving with a speed of 12.0 m/s, by conservation of momentum (Example 7–3). So the total kinetic energy afterward is

$$\tfrac{1}{2}(20{,}000\text{ kg})(12.0\text{ m/s})^2 = 1.44 \times 10^6\text{ J}.$$

Hence the energy transformed to other forms is

$$(2.88 \times 10^6\text{ J}) - (1.44 \times 10^6\text{ J}) = 1.44 \times 10^6\text{ J},$$

which is just half the original kinetic energy.

Markscheme-Topic 2: Mechanics

1. C
2. A
3. B
4. B
5. A
6. A
7. B
8. C
9. C
10. C
11. A
12. C
13. B
14. D
15. A
16. A
17. A
18. D
19. A
20. B
21. C
22. D
23. C
24. D
25. B
26. B
27. D
28. B
29. C
30. D
31. B
32. C
33. C
34. A
35. B
36. B
37. C
38. A

39. B

40. D

41. C

42. D

43. A

44. C

45. B

46. D

47. A

48. A

49. B

50. A

51. A

52. B

53. D

54. C

55. C

56. C

57. D

58. A

59. B

60. C

61. B

62. C

Short answer questions

1. Linear motion

(a) change in velocity / rate of change of velocity;
per unit time / with time; *(ratio idea essential to award this mark)* 2

(b) (i) acceleration is constant / uniform; 1

(ii) $t = \frac{2s}{(u+v)}$ and $t = \frac{(v-u)}{a}$;

clear working to obtain $v^2 = u^2 + 2as$; 2

(c) (i) 1.96 = $\frac{1}{2}$? 9.81 ? t^2;
t = 0.632s; 2

(ii) time to fall (1.96 + 0.12)m is 0.651s;
shutter open for 0.019s; 2
If the candidate gives a one significant digit answer treat it as an SD-1.
*Award **[0]** if the candidate uses s = $\frac{1}{2}$ at^2 and s = 12cm.*

[9]

2

2. (a) (i) $h = \dfrac{v^2}{2g}$;

to give $h = 3.2$ m; 2

(ii) 0.80 s; 1

(b) time to go from top of cliff to the sea = 3.0 – 1.6 = 1.4 s;

recognize to use $s = ut + \frac{1}{2}at^2$ with correct substitution,

$s = 8.0 \times 1.4 + 5.0 \times (1.4)^2$;

to give $s = 21$ m; 3

Answers might find the speed with which the stone hits the sea from $v = u + at$, (42 m s^{-1}) and then use $v^2 = u^2 + 2as$.

[6]

3. (a) $v_V = 8.0 \sin 60 = 6.9$ m s^{-1};

$h = \dfrac{v^2}{2g}$;

to give $h = 2.4$ m ; 3

*Award **[1]** if $v = 8.0$ m s^{-1} to get $h = 3.2$ m is used.*

(b) $v_H = 8.0 \cos 60$;

range = $v_H t = 8.0 \cos 60 \times 3 = 12$ m; 2

*Award **[1]** if $v = 8.0$ m s^{-1} to get $R = 2.4$ m is used.*

[5]

4. (a) use $\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2$ ***or*** some statement that conservation of energy is used;

$= \frac{1}{2}(15)^2 + 700 = \frac{1}{2}V^2$;

to give $V = 40\text{ms}^{-1}$ 2

(b) appreciate that the horizontal velocity remains unchanged;

so that $\theta = \cos^{-1}\dfrac{15}{40} = 68°$; 2

Accept alternative methods of solutions for (a) and (b) based on vertical component of velocity calculations and vector addition of components.

[4]

5. (a) (i) $v_x = \dfrac{40}{5.0} = 8.0$ m s^{-1} 1

Accept use of other values leading to the same answer.

(ii) $v_y = 0 = u_{0y} - 10 \times 2.5$;
$u_{0y} = 25 \text{ m s}^{-1}$; 2
Accept use of other values leading to the same answer.

(iii) the x and y components of displacement at 3.0 s are 24 m, 30 m;
so the magnitude is $\sqrt{24^2 + 30^2}$ = 38 m 2

(b) maximum height reached is less;
asymmetric with shorter range; 2

[7]

6. (a) zero; 1

(b) resultant vertical force from ropes = (2.15 ? 10^3 – weight) = 237N;
equating their result to $2T \sin 50$;
ie $2T \sin 50 = 237$
calculation to give T = 154.7N ? 150N; 3
Accept any value of tension from 130 N to 160 N. Award ***[2]*** *for missing factor of 2 but otherwise correct ie 309 N.*

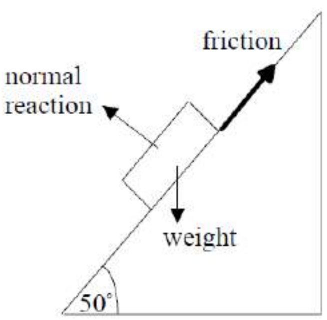
(c) correct substitution into $F = ma$;
to give $a = \dfrac{237}{1.95 \times 10^2} = 1.21 \text{ms}^{-2}$; 2
Watch for ecf.
NB *Depending on value of g answer will vary from 1.0(3) ms*$^{-2}$ *to 1.2(3) ms*$^{-2}$ *all of which are acceptable.*

(d) statement that air friction increases with increased speed seen / implied;
in 10 seconds friction goes from 0 N to 237 N / force increases from zero until it equals the net upward accelerating force; 2

[8]

7.

(a)



[1] for each correctly drawn and named force ***[3 max]***

(b) (i) calculation of acceleration from $a = \frac{2s}{t^2}$; *[1]*

to give $a = 2.47\ \text{m s}^{-2}$; *[1]*

[2 max]

(ii) component of weight down the plane $= M\,g \sin 50^\circ$ *[1]*

$= 7.51M$ *[1]*

(Do not penalise for omission of unit)

[2 max]

(c) $F = \mu_k N$; *[1]*

$= \mu_k Mg \cos 50^\circ = 6.31 M\mu_k$; *[1]*

(Do not penalise for omission of unit)

[2 max]

(d) accelerating force $= M(7.51 - 6.31\mu_k)$; *[1]*

$= M \times 2.47$ (mass × acceleration) *[1]*

to give $\mu_k = 0.80$; *[1]*

[3 max]

(e) recognise that $\mu_s = \tan\theta$; *[1]*

to give $\mu_s = 0.84$; *[1]*

[2 max] **[14]**

8.

(a) nature of the surfaces; normal reaction; relative motion of the surfaces; ***[2 max]***

(b) friction is the frictional force between an object and a surface / two surfaces;
static friction is (the frictional force) when the object/surfaces are at rest;
dynamic friction is(the frictional force) when the object is sliding / one of the surfaces is sliding / moving with respect to the other;
some additional comment *e.g.* friction varies from zero to maximum / maximum value of static friction always greater than kinetic friction; ***[3 max]***
*Award **[1 max]** for an answer such as "friction force on an object at rest and friction force on a moving object". Some appreciation that it is friction between two surfaces is required.*

(c) $\mu_s = \left(\frac{7.2}{12}\right) = 0.60$; *[1]*

(d) it will accelerate;
since the coefficient of dynamic friction is less than coefficient of static friction;
therefore, frictional force acting is less than 7.2 N / a net force greater than zero acting on the block; *[3]*
*Award **[0]** for a bald statement or incorrect reasoning.*

[9]

9. (a) mass × velocity; 1

(b) (i) momentum before = 800 × 5 = 4 000 N s;
momentum after = 2 000v;
conservation of momentum gives $v = 2.0\ \text{m s}^{-1}$; 3

(ii) KE before = 400 × 25 = 10 000 J   KE after = 1 000 × 4 = 4 000 J;
loss in KE = 6 000 J; 2

(c) transformed / changed into;
heat (internal energy) (and sound); 2

Do not accept "deformation of trucks".

[8]

10. (a) ***Note**: for part (i) and (ii) the answers in brackets are those arrived at if 19.3 is used as the value for the height.*

(i) height raised = 30 sin 40 = 19 m;
gain in PE = mgh = 700 × 19 = 1.3×10^4 J (1.4×10^4 J); 2

(ii) $48 \times 1.3 \times 10^4$ J = 6.2×10^5 J (6.7×10^5 J); 1

(iii) the people stand still / don't walk up the escalator
their average weight is 700 N / ignore any gain in KE of the people; 1 max

(b) power required = $\frac{(6.2 \times 10^5)}{60}$ = 10 kW (11kW);

$Eff = \frac{P_{\text{out}}}{P_{\text{in}}}$, $P_{\text{in}} = \frac{P_{\text{out}}}{Eff}$;

P_{in} = 14 kW (16 kW); 3

[7]

11. (a) momentum of object = $2 \times 10^3 \times 6.0$;
momentum after collision = $2.4 \times 10^3 \times v$;
use conservation of momentum, $2 \times 10^3 \times 6.0 = 2.4 \times 10^3 \times v$;
to get $v = 5.0$ m s^{-1}; 4

*Award **[2 max]** for mass after collision = 400 kg and v = 30 m s^{-1}.*

(b) KE of object and bar + change in PE = $1.2 \times 10^3 \times 25 + 2.4 \times 10^3 \times 10 \times 0.7533$;
use $\Delta E = Fd$, $4.8 \times 10^4 = F \times 0.75$;
to give F = 64 kN;

*Award **[2 max]** if PE missed F = 40 kN.*

or

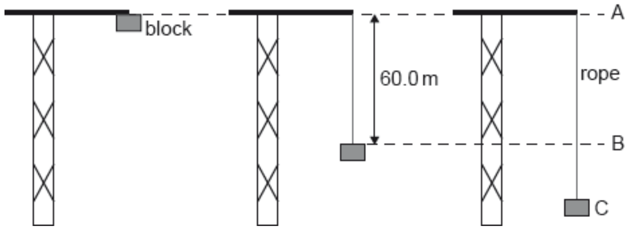
$a = \frac{v^2}{2s}$;
$F - mg = ma$;
to give F = 64kN; 3

*Award **[2 max]** if mg missed.*

[7]

Topic 2 (New) *[90 marks]*

An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

1a. At position B the rope starts to extend. Calculate the speed of the block at position B. *[2 marks]*

Markscheme

use of conservation of energy

OR

$v^2 = u^2 + 2as$

v = «$\sqrt{2 \times 60.0 \times 9.81}$» = 34.3 «$ms^{-1}$»

[2 marks]

At position C the speed of the block reaches zero. The time taken for the block to fall between B and C is 0.759 s. The mass of the block is 80.0 kg.

1b. Determine the magnitude of the average resultant force acting on the block between B and C. *[2 marks]*

Markscheme

use of impulse $F_{ave} \times \Delta t = \Delta p$

OR

use of $F = ma$ with average acceleration

OR

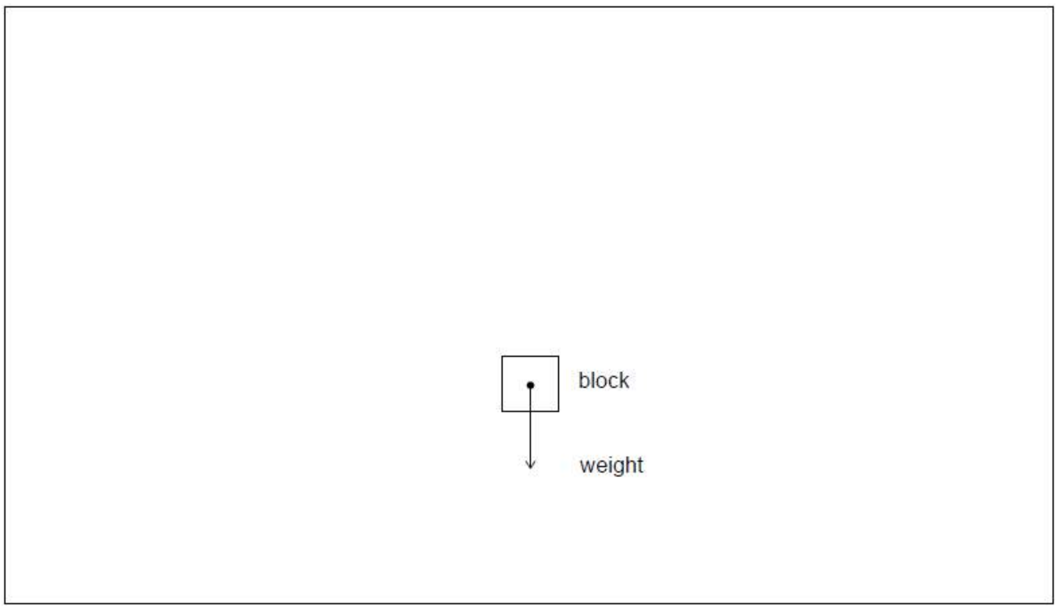
$F = \frac{80.0 \times 34.3}{0.759}$

3620«N»

Allow ECF from (a).

[2 marks]

1c. Sketch on the diagram the average resultant force acting on the block between B and C. The arrow on the diagram represents the weight of the block. *[2 marks]*



Markscheme

upwards

clearly longer than weight

For second marking point allow ECF from (b)(i) providing line is upwards.

[2 marks]

1d. Calculate the magnitude of the average force exerted by the rope on the block between B and C. *[2 marks]*

Markscheme

$3620 + 80.0 \times 9.81$

4400 «N»

Allow ECF from (b)(i).

[2 marks]

For the rope and block, describe the energy changes that take place

1e. between A and B. *[1 mark]*

Markscheme

(loss in) gravitational potential energy (of block) into kinetic energy (of block)

Must see names of energy (gravitational potential energy and kinetic energy) – Allow for reasonable variations of terminology (eg energy of motion for KE).

[1 mark]

1f. between B and C. *[1 mark]*

Markscheme

(loss in) gravitational potential and kinetic energy of block into elastic potential energy of rope

See note for 1(c)(i) for naming convention.

Must see either the block or the rope (or both) mentioned in connection with the appropriate energies.

[1 mark]

1g. The length reached by the rope at C is 77.4 m. Suggest how energy considerations could be used to determine the elastic constant of the rope. *[2 marks]*

Markscheme

k can be determined using EPE = $\frac{1}{2}kx^2$

correct statement or equation showing

GPE at A = EPE at C

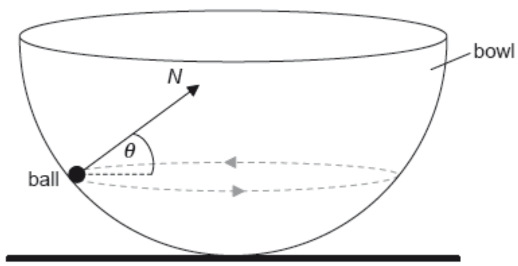
OR

(GPE + KE) at B = EPE at C

Candidate must clearly indicate the energy associated with either position A or B for MP2.

[2 marks]

A small ball of mass m is moving in a horizontal circle on the inside surface of a frictionless hemispherical bowl.



The normal reaction force N makes an angle θ to the horizontal.

2a. State the direction of the resultant force on the ball. *[1 mark]*

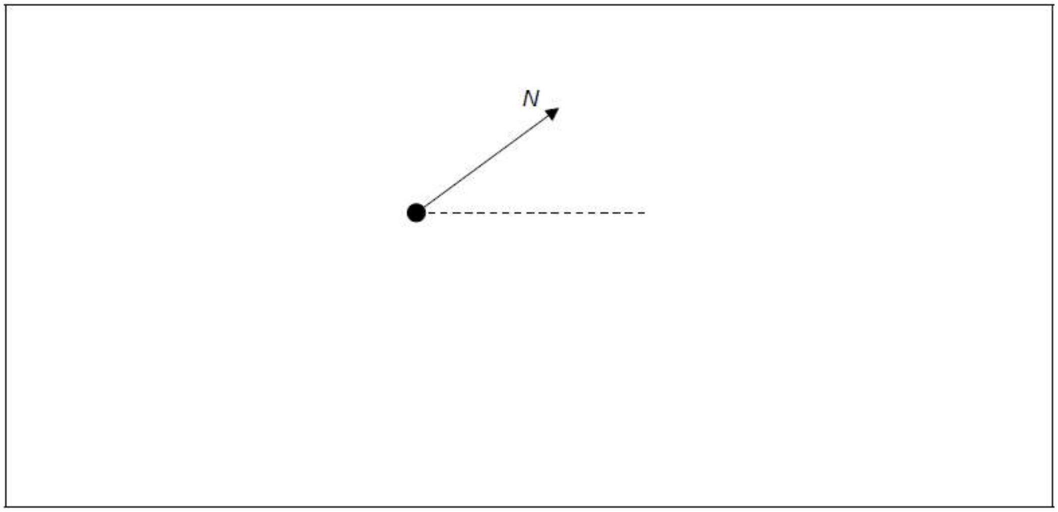
Markscheme

towards the centre «of the circle» / horizontally to the right

Do not accept towards the centre of the bowl

[1 mark]

2b. On the diagram, construct an arrow of the correct length to represent the weight of the ball. *[2 marks]*



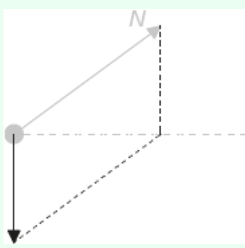
Markscheme

downward vertical arrow of any length

arrow of correct length

Judge the length of the vertical arrow by eye. The construction lines are not required. A label is not required

eg:



[2 marks]

2c. Show that the magnitude of the net force *F* on the ball is given by the following equation. *[3 marks]*

$$F = \frac{mg}{\tan\theta}$$

Markscheme

ALTERNATIVE 1

$F = N\cos\theta$

$mg = N\sin\theta$

dividing/substituting to get result

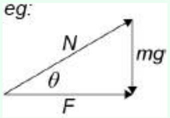
ALTERNATIVE 2

right angle triangle drawn with F, N and W/mg labelled

angle correctly labelled and arrows on forces in correct directions

correct use of trigonometry leading to the required relationship

eg:



$\tan\theta = \frac{O}{A} = \frac{mg}{F}$

[3 marks]

2d. The radius of the bowl is 8.0 m and $\theta = 22°$. Determine the speed of the ball. *[4 marks]*

Markscheme

$\frac{mg}{\tan\theta} = m\frac{v^2}{r}$

$r = R\cos\theta$

$v = \sqrt{\frac{gR\cos^2\theta}{\sin\theta}} / \sqrt{\frac{gR\cos\theta}{\tan\theta}} / \sqrt{\frac{9.81 \times 8.0\cos 22}{\tan 22}}$

$v = 13.4/13$ «ms^{-1}»

*Award **[4]** for a bald correct answer*

*Award **[3]** for an answer of 13.9/14 «ms^{-1}». MP2 omitted*

[4 marks]

2e. Outline whether this ball can move on a horizontal circular path of radius equal to the radius of the bowl. *[2 marks]*

Markscheme

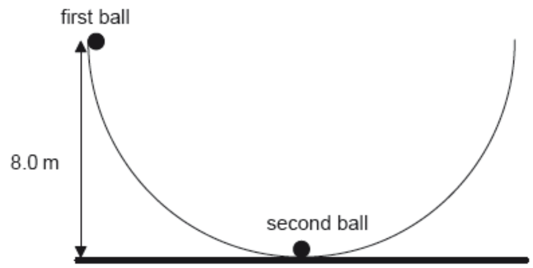
there is no force to balance the weight/N is horizontal

so no / it is not possible

Must see correct justification to award MP2

[2 marks]

2f. A second identical ball is placed at the bottom of the bowl and the first ball is displaced so that its height from the horizontal is equal to 8.0 m. *[3 marks]*



The first ball is released and eventually strikes the second ball. The two balls remain in contact. Determine, in m, the maximum height reached by the two balls.

Markscheme

speed before collision v = «$\sqrt{2gR}$ =» 12.5 «ms^{-1}»

«from conservation of momentum» common speed after collision is $\frac{1}{2}$ initial speed «$v_c = \frac{12.5}{2}$ = 6.25 ms^{-1}»

h = «$\frac{v_c{}^2}{2g} = \frac{6.25^2}{2 \times 9.81}$» 2.0 «m»

Allow 12.5 from incorrect use of kinematics equations

*Award **[3]** for a bald correct answer*

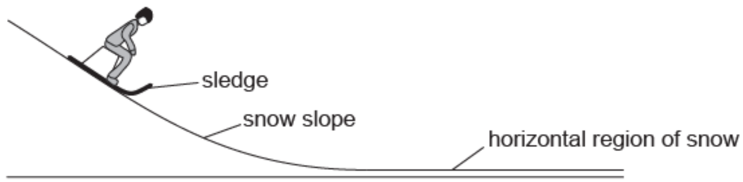
*Award **[0]** for mg(8) = 2mgh leading to h = 4 m if done in one step.*

Allow ECF from MP1

Allow ECF from MP2

[3 marks]

A girl on a sledge is moving down a snow slope at a uniform speed.



3a. Draw the free-body diagram for the sledge at the position shown on the snow slope. *[2 marks]*

Markscheme

arrow vertically downwards labelled weight «of sledge and/or girl»/ *W*/mg/gravitational force/F_g/ $F_{gravitational}$ ***AND*** arrow perpendicular to the snow slope labelled reaction force/*R*/normal contact force/N/ F_N

friction force/*F*/*f* acting up slope «perpendicular to reaction force»

Do not allow G/g/"gravity".

Do not award MP1 if a "driving force" is included.

Allow components of weight if correctly labelled.

Ignore point of application or shape of object.

Ignore "air resistance".

Ignore any reference to "push of feet on sledge".

Do not award MP2 for forces on sledge on horizontal ground

The arrows should contact the object

3b. After leaving the snow slope, the girl on the sledge moves over a horizontal region of snow. Explain, with reference to the physical origin of the forces, why the vertical forces on the girl must be in equilibrium as she moves over the horizontal region. *[3 marks]*

Markscheme

gravitational force/weight from the Earth «downwards»

reaction force from the sledge/snow/ground «upwards»

no vertical acceleration/remains in contact with the ground/does not move vertically as there is no resultant vertical force

Allow naming of forces as in (a)

Allow vertical forces are balanced/equal in magnitude/cancel out

3c. When the sledge is moving on the horizontal region of the snow, the girl jumps off the sledge. The girl has no horizontal velocity after the jump. The velocity of the sledge immediately after the girl jumps off is 4.2 m s^{-1}. The mass of the girl is 55 kg and the mass of the sledge is 5.5 kg. Calculate the speed of the sledge immediately before the girl jumps from it. *[2 marks]*

Markscheme

mention of conservation of momentum

OR

5.5 x 4.2 = (55 + 5.5) «v»

0.38 «m s^{-1}»

Allow p=p´ or other algebraically equivalent statement

*Award **[0]** for answers based on energy*

3d. The girl chooses to jump so that she lands on loosely-packed snow rather than frozen ice. Outline why she chooses to land on the snow. *[3 marks]*

Markscheme

same change in momentum/impulse

the time taken «to stop» would be greater «with the snow»

$F = \frac{\Delta p}{\Delta t}$ therefore *F* is smaller «with the snow»

OR

force is proportional to rate of change of momentum therefore *F* is smaller «with the snow»

Allow reverse argument for ice

The sledge, without the girl on it, now travels up a snow slope that makes an angle of 6.5˚ to the horizontal. At the start of the slope, the speed of the sledge is 4.2 m s^{-1}. The coefficient of dynamic friction of the sledge on the snow is 0.11.

3e. Show that the acceleration of the sledge is about –2 m s^{-2}. *[3 marks]*

Markscheme

«friction force down slope» = *μmg* cos(6.5) = «5.9 N»

«component of weight down slope» = *mg* sin(6.5) «= 6.1 N»

«so $a = \frac{F}{m}$» acceleration = $\frac{12}{5.5}$ = 2.2 «m s^{-2}»

Ignore negative signs

Allow use of g = 10 m s^{-2}

3f. Calculate the distance along the slope at which the sledge stops moving. Assume that the coefficient of dynamic friction is constant. *[2 marks]*

Markscheme

correct use of kinematics equation

distance = 4.4 ***or*** 4.0 «m»

Alternative 2

KE lost=work done against friction + GPE

distance = 4.4 ***or*** 4.0 «m»

Allow ECF from (e)(i)

*Allow **[1 max]** for GPE missing leading to 8.2 «m»*

3g. The coefficient of static friction between the sledge and the snow is 0.14. Outline, with a calculation, the subsequent motion of the sledge. *[2 marks]*

Markscheme

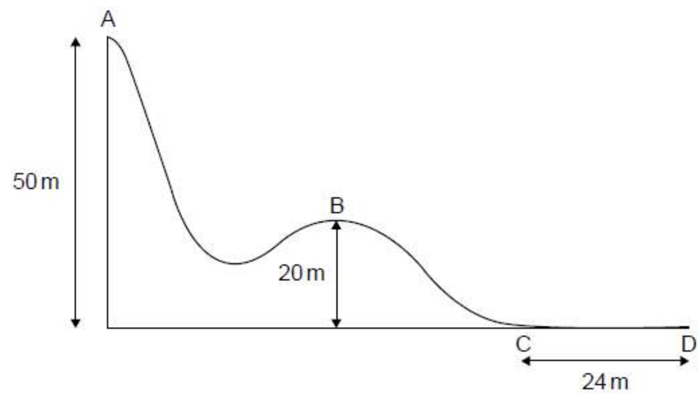
calculates a maximum value for the frictional force = « μR=» 7.5 «N»

sledge will not move as the maximum static friction force is greater than the component of weight down the slope

Allow correct conclusion from incorrect MP1

Allow 7.5 > 6.1 so will not move

The diagram below shows part of a downhill ski course which starts at point A, 50 m above level ground. Point B is 20 m above level ground.



A skier of mass 65 kg starts from rest at point A and during the ski course some of the gravitational potential energy transferred to kinetic energy.

4a. From A to B, 24 % of the gravitational potential energy transferred to kinetic energy. Show that the velocity at B is 12 m s^{-1}. *[2 marks]*

Markscheme

$\frac{1}{2}v^2 = 0.24\,\mathrm{gh}$

$v = 11.9$ «m s^{-1}»

Award GPE lost = 65 × 9.81 × 30 = «19130 J»

Must see the 11.9 value for MP2, not simply 12.

Allow g = 9.8 ms^{-2}.

4b. Some of the gravitational potential energy transferred into internal energy of the skis, slightly increasing their temperature. Distinguish between internal energy and temperature. *[2 marks]*

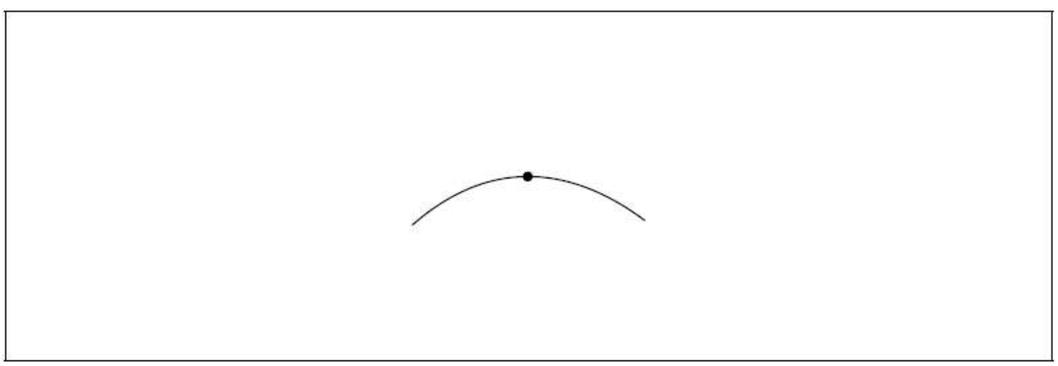
Markscheme

internal energy is the total KE «and PE» of the molecules/particles/atoms in an object

temperature is a measure of the average KE of the molecules/particles/atoms

*Award **[1 max]** if there is no mention of molecules/particles/atoms.*

4c. The dot on the following diagram represents the skier as she passes point B. Draw and label the vertical forces acting on the skier. *[2 marks]*



Markscheme

arrow vertically downwards from dot labelled weight/W/mg/gravitational force/F_g/$F_{gravitational}$ ***AND*** arrow vertically upwards from dot labelled reaction force/R/normal contact force/N/F_N

W > R

Do not allow gravity.
Do not award MP1 if additional 'centripetal' force arrow is added.
Arrows must connect to dot.
Ignore any horizontal arrow labelled friction.
Judge by eye for MP2. Arrows do not have to be correctly labelled or connect to dot for MP2.

4d. The hill at point B has a circular shape with a radius of 20 m. Determine whether the skier will lose contact with the ground at point B. *[3 marks]*

Markscheme

ALTERNATIVE 1

recognition that centripetal force is required / $\frac{mv^2}{r}$ seen

= 468 «N»

W/640 N (weight) is larger than the centripetal force required, so the skier does not lose contact with the ground

ALTERNATIVE 2

recognition that centripetal acceleration is required / $\frac{v^2}{r}$ seen

a = 7.2 «ms^{-2}»

g is larger than the centripetal acceleration required, so the skier does not lose contact with the ground

ALTERNATIVE 3

recognition that to lose contact with the ground centripetal force ≥ weight

calculation that v ≥ 14 «ms^{-1}»

comment that 12 «ms^{-1}» is less than 14 «ms^{-1}» so the skier does not lose contact with the ground

ALTERNATIVE 4

recognition that centripetal force is required / $\frac{mv^2}{r}$ seen

calculation that reaction force = 172 «N»

reaction force > 0 so the skier does not lose contact with the ground

Do not award a mark for the bald statement that the skier does not lose contact with the ground.

4e. The skier reaches point C with a speed of 8.2 m s^{-1}. She stops after a distance of 24 m at point D. *[3 marks]*

Determine the coefficient of dynamic friction between the base of the skis and the snow. Assume that the frictional force is constant and that air resistance can be neglected.

Markscheme

ALTERNATIVE 1
$0 = 8.2^2 + 2 \times a \times 24$ therefore $a =$ «–»1.40 «m s^{-2}»

friction force = $ma = 65 \times 1.4 = 91$ «N»

coefficient of friction = $\frac{91}{65 \times 9.81}$ = 0.14

ALTERNATIVE 2
$KE = \frac{1}{2}mv^2 = 0.5 \times 65 \times 8.2^2 = 2185$ «J»

friction force = KE/distance = 2185/24 = 91 «N»

coefficient of friction = $\frac{91}{65 \times 9.81}$ = 0.14

Allow ECF from MP1.

At the side of the course flexible safety nets are used. Another skier of mass 76 kg falls normally into the safety net with speed 9.6 m s^{-1}.

4f. Calculate the impulse required from the net to stop the skier and state an appropriate unit for your answer. *[2 marks]*

Markscheme

«76 × 9.6»= 730
Ns ***OR*** kg ms^{-1}

4g. Explain, with reference to change in momentum, why a flexible safety net is less likely to harm the skier than a rigid barrier. *[2 marks]*

Markscheme

safety net extends stopping time

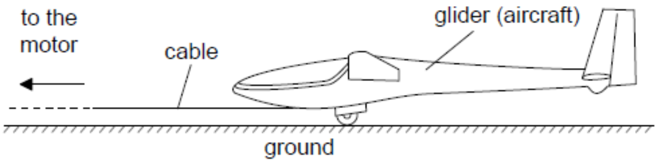
$F = \frac{\Delta p}{\Delta t}$ therefore F is smaller «with safety net»

OR

force is proportional to rate of change of momentum therefore F is smaller «with safety net»

Accept reverse argument.

A glider is an aircraft with no engine. To be launched, a glider is uniformly accelerated from rest by a cable pulled by a motor that exerts a horizontal force on the glider throughout the launch.



5a. The glider reaches its launch speed of 27.0 m s^{-1} after accelerating for 11.0 s. Assume that the glider moves horizontally until it leaves the ground. Calculate the total distance travelled by the glider before it leaves the ground. *[2 marks]*

Markscheme

correct use of kinematic equation/equations

148.5 ***or*** 149 ***or*** 150 «m»

Substitution(s) must be correct.

5b. The glider and pilot have a total mass of 492 kg. During the acceleration the glider is subject to an average resistive force of 160 N. Determine the average tension in the cable as the glider accelerates. *[3 marks]*

Markscheme

$a = \frac{27}{11}$ ***or*** 2.45 «m s^{-2}»

$F - 160 = 492 \times 2.45$

1370 «N»

Could be seen in part (a).
*Award **[0]** for solution that uses a = 9.81 m s^{-2}*

5c. The cable is pulled by an electric motor. The motor has an overall efficiency of 23 %. Determine the average power input to the motor. *[3 marks]*

Markscheme

ALTERNATIVE 1

«work done to launch glider» = 1370 x 149 «= 204 kJ»

«work done by motor» $= \frac{204 \times 100}{23}$

«power input to motor» $= \frac{204 \times 100}{23} \times \frac{1}{11} = 80$ ***or*** 80.4 ***or*** 81 k«W»

ALTERNATIVE 2

use of average speed 13.5 m s^{-1}

«useful power output» = force x average speed «= 1370 x 13.5»

power input = «$1370 \times 13.5 \times \frac{100}{23}$ =» 80 ***or*** 80.4 ***or*** 81 k«W»

ALTERNATIVE 3

work required from motor = KE + work done against friction «
$= 0.5 \times 492 \times 27^2 + (160 \times 148.5)$» = 204 «kJ»

«energy input» $= \frac{\text{work required from motor} \times 100}{23}$

power input $= \frac{883000}{11} = 80.3$ k«W»

*Award **[2 max]** for an answer of 160 k«W».*

5d. The cable is wound onto a cylinder of diameter 1.2 m. Calculate the angular velocity of the cylinder at the instant when the glider has a speed of 27 m s^{-1}. Include an appropriate unit for your answer. *[2 marks]*

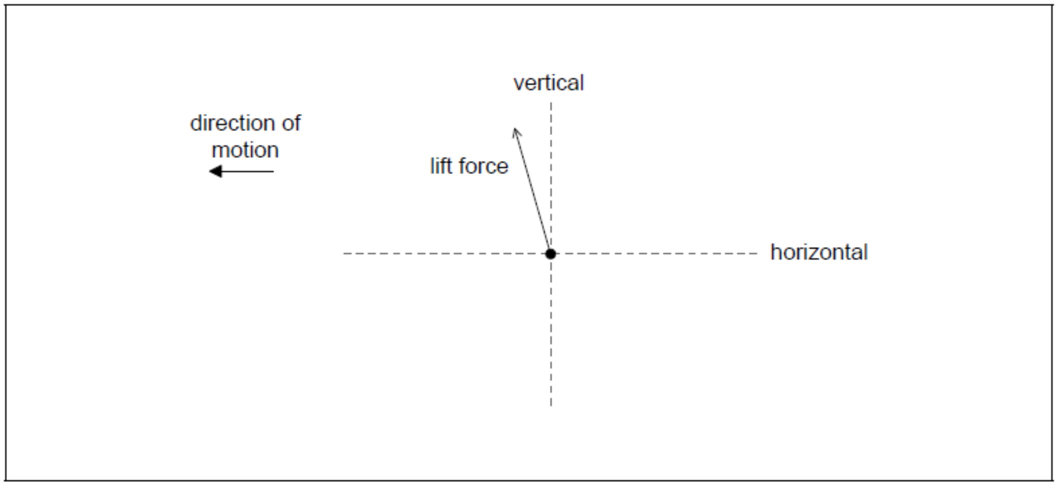
Markscheme

$\omega =$ «$\frac{v}{r} =$» $\frac{27}{0.6} = 45$

rad s^{-1}

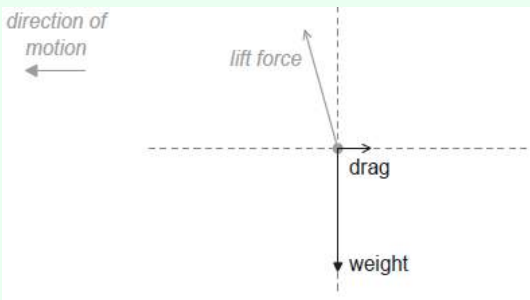
Do not accept Hz.
*Award **[1 max]** if unit is missing.*

5e. After takeoff the cable is released and the unpowered glider moves horizontally at constant speed. The wings of the glider provide a lift force. The diagram shows the lift force acting on the glider and the direction of motion of the glider. *[2 marks]*



Draw the forces acting on the glider to complete the free-body diagram. The dotted lines show the horizontal and vertical directions.

Markscheme



drag correctly labelled and in correct direction

weight correctly labelled and in correct direction ***AND*** no other incorrect force shown

*Award **[1 max]** if forces do not touch the dot, but are otherwise OK.*

5f. Explain, using appropriate laws of motion, how the forces acting on the glider maintain it in level flight. *[2 marks]*

Markscheme

name Newton's first law

vertical/all forces are in equilibrium/balanced/add to zero
OR
vertical component of lift mentioned

as equal to weight

5g. At a particular instant in the flight the glider is losing 1.00 m of vertical height for every 6.00 m that it goes forward horizontally. At this instant, the horizontal speed of the glider is 12.5 m s^{-1}. Calculate the **velocity** of the glider. Give your answer to an appropriate number of significant figures. *[3 marks]*

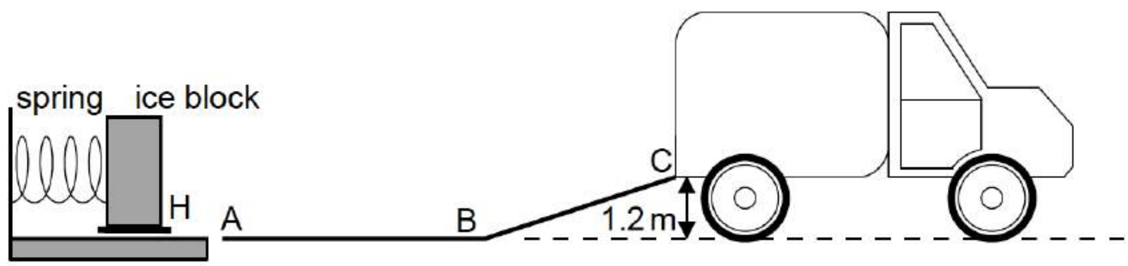
Markscheme

any speed and any direction quoted together as the answer

quotes their answer(s) to 3 significant figures

speed = 12.7 m s^{-1} ***or*** direction = 9.46° ***or*** 0.165 rad «below the horizontal» ***or*** gradient of $-\frac{1}{6}$

A company designs a spring system for loading ice blocks onto a truck. The ice block is placed in a holder H in front of the spring and an electric motor compresses the spring by pushing H to the left. When the spring is released the ice block is accelerated towards a ramp ABC. When the spring is fully decompressed, the ice block loses contact with the spring at A. The mass of the ice block is 55 kg.



Assume that the surface of the ramp is frictionless and that the masses of the spring and the holder are negligible compared to the mass of the ice block.

6a. (i) The block arrives at C with a speed of 0.90ms $^{-1}$. Show that the elastic energy stored in the spring is 670J. *[4 marks]*

(ii) Calculate the speed of the block at A.

Markscheme

(i)

$\ll E_{\text{el}} = \gg \frac{1}{2}mv^2 + mgh$

OR

«E_{el}=» E_P+E_K

$\ll E_{\text{el}} = \gg \frac{1}{2} \times 55 \times 0.90^2 + 55 \times 9.8 \times 1.2$

OR

669 J

«E_{el} = 669 ≈ 670J»

*Award **[1 max]** for use of g=10Nkg^{-1}, gives 682 J.*

(ii)

$\frac{1}{2} \times 55 \times v^2 = 670\text{J}$

$v = \ll \sqrt{\frac{2\times 670}{55}} = \gg 4.9\text{ms}^{-1}$

If 682J used, answer is 5.0ms^{-1}.

6b. Describe the motion of the block *[3 marks]*

(i) from A to B with reference to Newton's first law.

(ii) from B to C with reference to Newton's second law.

Markscheme

(i)

no force/friction on the block, hence constant motion/velocity/speed

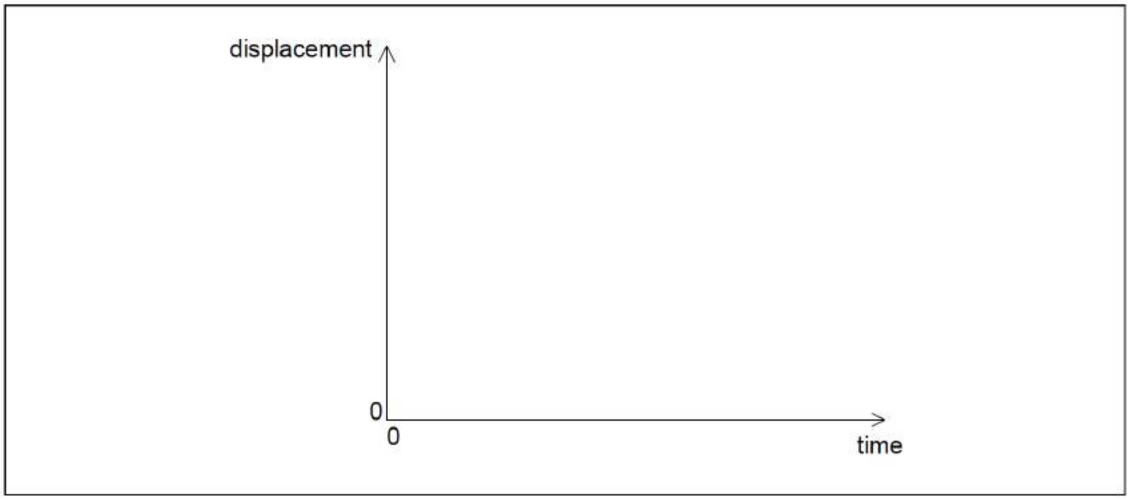
(ii)

force acts on block ***OR*** gravity/component of weight pulls down slope

velocity/speed decreases ***OR*** it is slowing down ***OR*** it decelerates

Do not allow a bald statement of "N2" or "F = ma" for MP1.
Treat references to energy as neutral.

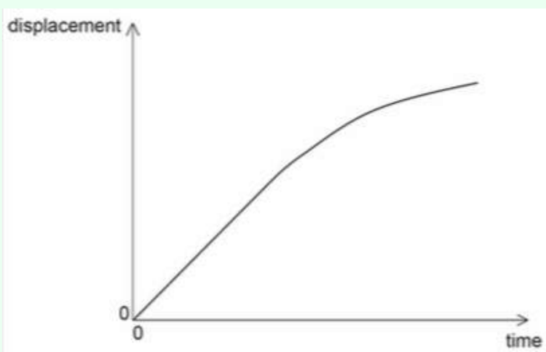
6c. On the axes, sketch a graph to show how the displacement of the block varies with time from A to C. (You do not have to put numbers on the axes.) *[2 marks]*



Markscheme

straight line through origin for at least one-third of the total length of time axis **covered by candidate line**

followed by curve with decreasing positive gradient



Ignore any attempt to include motion before A.

Gradient of curve must always be less than that of straight line.

6d. The spring decompression takes 0.42s. Determine the average force that the spring exerts on the block. *[2 marks]*

Markscheme

$F \ll= \frac{\Delta p}{\Delta t} \gg= \frac{55\times4.9}{0.42}$

F=642≈640N

Allow ECF from (a)(ii).

6e. The electric motor is connected to a source of potential difference 120V and draws a current of 6.8A. The motor takes 1.5s to compress the spring. *[2 marks]*

Estimate the efficiency of the motor.

Markscheme

«energy supplied by motor =» 120 × 6.8 × 1.5 ***or*** 1224 J
OR
«power supplied by motor =» 120 × 6.8 ***or*** 816 W
e = 0.55 ***or*** 0.547 or 55% ***or*** 54.7%

Allow ECF from earlier results.

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